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Multicriteria ranking of inventory replenishment policies in the presence of uncertainty in customer demand

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Abstract

This paper considers the ranking of alternative replenishment policies for a single-location inventory when customer demand is confined to a single product. It is supposed that customer demand is uncertain and described by imprecise linguistic terms defined by fuzzy sets. For example, "demand is about *d*", or "demand is more or less between d_L and d_U", or more complex "demand is about *d*, but there is a possibility to be 2*d*", etc. Four alternative replenishment policies are considered in particular: (1) fixed cycle–fixed order quantity, (2) fixed cycle–variable order quantity, (3) variable review period–fixed order quantity, and (4) variable review period–variable order quantity. With each replenishment policy a number of criteria are associated. Typical criteria examined are: (a) average fill rate, (b) inventory holding cost per item delivered, (c) regularity of the replenishment orders with regard to time and quantity of ordering. Criteria values for each replenishment policy can be obtained either analytically or by using a simulation technique, or they are linguistic subjective judgements defined by fuzzy sets, like, for example, the values of criterion (c). In order to choose the best replenishment policy, a procedure for selecting a compromised optimistic–pessimistic solution is developed and applied. It enables the comparison of a finite number of alternative replenishment policies with respect to a number of criteria, simultaneously, when the criteria values are either cardinal or linguistic terms, and the relative weights of the criteria are vaguely expressed. An illustrative example is given. © 2001 Elsevier Science B.V. All rights reserved.

Keywords: Replenishment; Ranking; Uncertainty; Fuzzy set; Multicriteria analysis

1. Introduction

In this paper, we consider the basic questions arising in single-product, single-location dynamic inventory control: *when to order and how much to order*. The answers to these questions covers an important segment of inventory theory. Determination of the set of rules, according to which the filling of an inventory is made, referred to as replenishment policy (RP), is particularly a challenging problem in the case where an inventory operates in an uncertain environment. Generally, an *uncertain environment* implies that there exist various external sources and types of uncertainty, which influence customer demand for product and a replenishment process. For example, customer demand depends on pricing, appearance of competition, product obsolescence and other vaguely defined factors. Uncertainties in the replenishment process include procurement lead-time, quality control time from the moment when the order arrives until it replenishes the stock, etc.

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One may say that a real inventory always operates in the presence of random events, lack of evidence, lack of certainty of evidence, or simply in the presence of information imprecision. For example, very often uncertain customer demand is described by vague linguistic phrases such as: "demand is *about d* products per week", or "monthly demand is *much larger* than d_L", or "demand in the next period will be in the interval $[d_L, d_U]$ with a *high* degree of possibility, but there is a *moderate* degree of possibility that demand will be zero", etc. It is the contention of this paper that the sort of uncertainty that comes into play here is better represented by the notion of fuzziness than that of chance and probability [1].

In dealing with inventory replenishment the structural characteristics of RP are most often assumed to be known, but its control variables unknown. Usually, the structural characteristics of RP are taken to mean the following: (1) reorder cycle is fixed or variable, and (2) lot sizes ordered are fixed or variable. Depending on the type of RP, the control variables most often used are the length of the review period, safety or reorder level and order quantity.

Since Wilson's formulae which date back to 1930s, a considerable number of papers have been published, dealing with the problem of how to determine the control variables of different RPs [2–5]. The basic inventory control task has been stated as: for a given inventory environment and a proposed RP, determine the optimum control variables associated with RP under consideration. Under some reasonable assumptions about the inventory environment, the optimum control variables with respect to a single optimisation criterion can be determined. It can be done for inventories with deterministic, stochastic or even fuzzy demand [6]. However, in practice, experts often specify inventory control variables based on their experience, which is a good practice. When the inventory control variables are determined, whether optimal or not, a number of inventory performance measures can be calculated. They usually describe service level and cost characteristics and can be calculated analytically or by simulation.

The objective of this paper is to model a number of alternative RPs in the case where customer demand is uncertain and described by fuzzy sets [7], and to choose one among them. It is assumed that the RPs are completely specified, including all the control variables. The control variables for each RP are not necessarily optimal. Experts can also specify them, on the basis of practical good sense and judgement gained from experience.

In the authors' opinion, choosing from alternative RPs has to be made taking into account a number of criteria, representing many aspects that enter into the evaluation of each RP. The values of each criterion are either cardinal or linguistic expressions. Moreover, varying degrees of importance are attached to all the criteria. The relative importance of each criterion is usually given by a linguistic term, such as "very important", "low important", etc.

The paper is organised as follows. In Section 2 a multicriteria approach to selecting the best RP is given. In Section 3 a new algorithm for the selection of the compromised best RP with respect to a number of criteria, simultaneously, is described. Section 4 gives an illustrative example. The algorithm developed involves a comparison of one imprecise value with another imprecise value. A new procedure which compares imprecise values which are defined by discrete fuzzy sets, is presented in the Appendix.

2. Problem statement

2.1. Basic assumptions

Assumptions concerning inventory processes considered in this paper are as follows:

- Single location inventory is treated.
- Customer demand is confined to a single product.
- When demand exceeds the stock, unmet demand is backordered and delivered as soon as it becomes available.
- The inventory is replenished from an external source and replenishment quantities are received either promptly or with a given lead time.
- Customer demand is uncertain and it is vaguely expressed by some linguistic term, formally described by a fuzzy set. Two examples are given

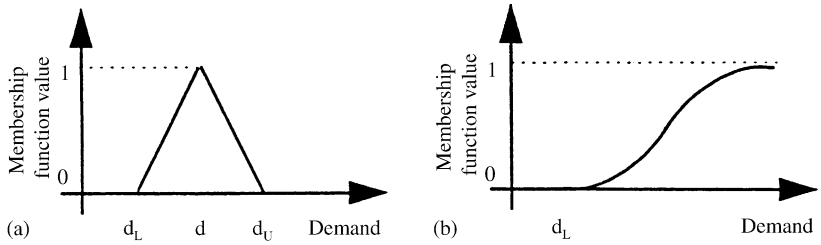

Fig. 1. Membership functions of fuzzy customer demand. (a) "demand is *about d*, but definitely not less than d_L and not greater than d_U". (b) "demand is *much larger* than d_L".

in Fig. 1. Of course, many linguistic expressions about customer demand can be interpreted by appropriate fuzzy sets.

2.2. Replenishment policies

A number of RPs are defined in the literature and exist in practice. In this paper we shall focus on four of them, that have most extensively been studied in inventory theory and most widely applied in practice.

(1) RP_1: Fixed cycle–fixed order quantity, where a fixed order quantity is placed periodically (Fig. 2(a)).
(2) RP_2: Fixed cycle–variable order quantity, where inventory records are reviewed periodically and the order quantity is determined in such a way as to bring the stock level to a predetermined order-up-to level (Fig. 2(b)).
(3) RP_3: Variable review period–fixed order quantity, where the periods between reordering may change and a fixed order quantity is placed when the stock falls below a predetermined reorder point (Fig. 2(c)).
(4) RP_4: Variable review period–variable order quantity, where both the periods between reordering and order quantities may change over time and order quantities bring the stock up to a predetermined order-up-to level (Fig. 2(d)).

2.3. Criteria

With each RP a number of performance measure criteria are associated. It is supposed that all the criteria that enter into the evaluation of RPs are relevant for each RP. Three criteria that represent an inventory service level, a cost aspect and a relationship with supplier side are typical, but not used exclusively. They are:

C_1: fill rate achieved over a given time horizon,
C_2: total holding cost per item demanded,
C_3: regularity of the replenishment orders with respect to time instant and quantities ordered.

The values of criteria C_1 and C_2 for each particular RP can be obtained analytically or by simulation and the value of criteria C_3 is a subjective judgement described by a linguistic expression. For example, the following linguistic expressions can be used: *very high regularity, high regularity, moderate regularity, low regularity*, and *very low regularity*. The membership functions of the corresponding discrete fuzzy sets are given on scale [1–5], Fig. 3. Let us add that it is, of course, possible to consider some other evaluative concepts which describe service performance or different types of inventory cost characteristics.

2.4. Relative importance of criteria

All the criteria usually do not have the same importance. The relative importance of each criterion is subjectively assessed by a weighting coefficient, which is supposed to be a vague linguistic expression. A discrete fuzzy set is associated with each vague linguistic expression. For example, the membership functions of the three primary fuzzy terms, namely *low importance, moderate importance* and *very high importance*, are given in Fig. 4. The

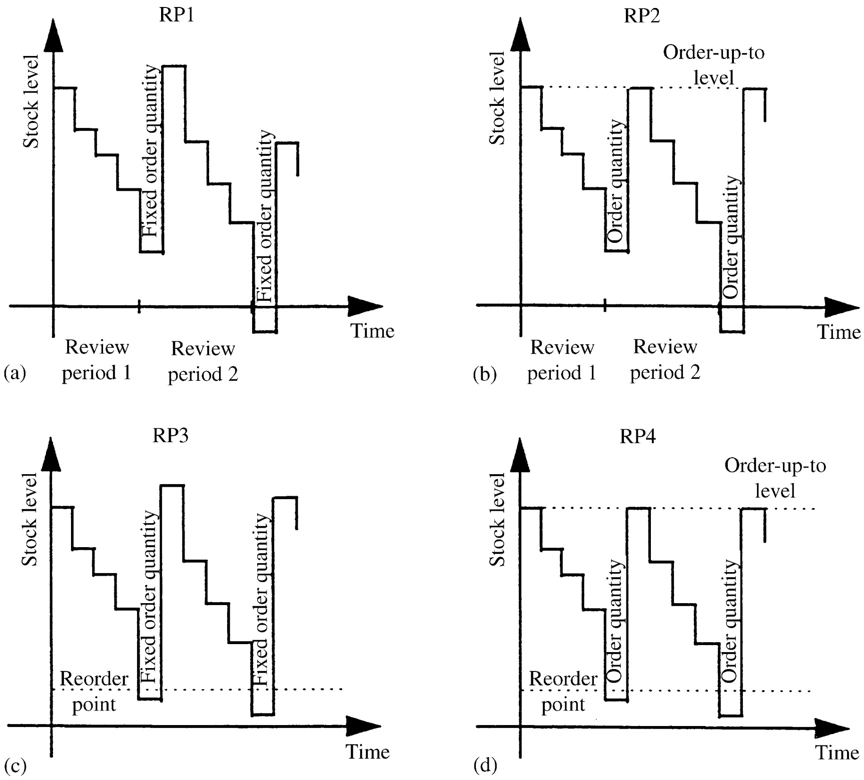

Fig. 2. Stock over time in different RPs. (a) Fixed review period, fixed order quantity. (b) Fixed review period, order-up-to level. (c) Variable review period, fixed order quantity. (d) Variable review period, order-up-to level.

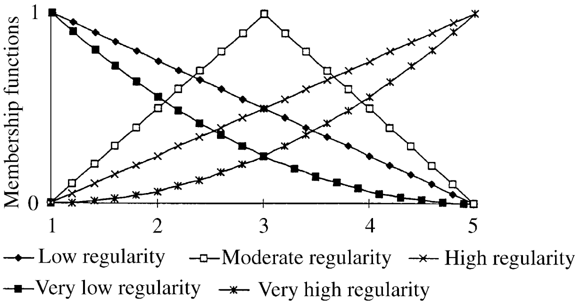

Fig. 3. Discrete fuzzy sets which represent regularity of replenishment orders with respect to time instant and quantities ordered.

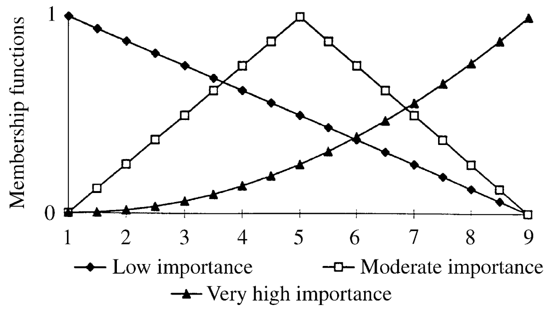

Fig. 4. Three fuzzy sets that describe relative importance of criteria.

discrete fuzzy sets are defined on the almost standard integer scale [1–9]. One could include in the consideration some additional fuzzy terms defined by linguistic hedges, such as *fairly* or *very, very*, e.g. *fairly low importance, very, very high importance*, etc.

2.5. Matrix form of the problem statement

Generally, the problem of selecting the best from among a finite number of RPs is the following: given I replenishment policies $\mathrm{RP}_1, \ldots, \mathrm{RP}_i, \ldots, \mathrm{RP}_I$ and K criteria $C_1, \ldots, C_k, \ldots, C_K$. The criteria are either of a benefit type (the larger the criterion value the better) or a cost type (the smaller the criterion value the better). Criteria values f_{ik}, $i \in I$, $k \in \mathscr{K}$, where I and $\mathscr{K}$ are the corresponding sets of indices, are arranged in an $I \times K$ matrix F. An additional row, at the bottom of F determines the type of each criterion. The values f_{ik} for each column $k \in \mathscr{K}$ are either cardinal or linguistic. With each criterion k, relative importance w_k as a vague linguistic expression is associated and given in the last row of F. The linguistic values of f_{ik} and w_k are defined by discrete fuzzy sets.

$$F = \begin{array}{c} \\ RP_1 \\ \vdots \\ RP_i \\ \vdots \\ RP_I \\ \\ \\ \end{array} \begin{array}{c} \begin{array}{ccccc} C_1 & \cdots & C_k & \cdots & C_K \end{array} \\ \left[\begin{array}{ccccc} & & \vdots & & \\ & & \vdots & & \\ \cdots & \cdots & f_{ik} & & \\ & & & & \\ & & & & \end{array}\right] \\ \begin{array}{ccccc} \max(\min) & \ldots & \max(\min) & \ldots & \max(\min) \\ w_1 & \cdots & w_k & \cdots & w_K \end{array} \end{array}.$$

The problem is to rank all RPs and choose the best RP* with respect to all criteria, simultaneously, taking into account the type of each criterion and its relative importance.

3. Algorithm for the selection of the best RP

A new algorithm for multicriteria selection of the best RP* is proposed. It is based on an adaptation of a Hurwitz approach for selecting a combined optimistic–pessimistic solution [8]. This means that an RP* with the specific compromised characteristics has to be selected, its best criteria value has to be very high – possibly the highest, and at the same time its weakest criteria value has to be not bad, possibly the least bad. The algorithm developed for selecting such an RP* has the following steps:

Step 1: Transform all the cardinal criteria values f_{ik} into r_{ik} defined on a common scale [0, 1] by applying linear transformations:

(1) for a benefit type criterion $k \in \mathscr{K}$

$$r_{ik} = \frac{f_{ik}}{f_k^{\max}},$$

(2) for a cost-type criterion $k \in \mathscr{K}$

$$r_{ik} = 1 - \frac{f_{ik} - f_k^{\min}}{f_k^{\max}},$$

where $f_k^{\min} = \min_i f_{ik}$ and $f_k^{\max} = \max_i f_{ik}$.

Step 2: Transform all the linguistic criteria values f_{ik}, into degrees of belief b_{ik} expressed on a common scale [0, 1] by applying a fuzzy set comparison method given in the Appendix, i.e.:

(1) for a benefit-type criterion $k \in \mathscr{K}$, find the degree of belief b_{ik} that f_{ik} is greater or equal to all other f_{jk}, $j \in I \backslash i$,
(2) for a cost-type criterion $k \in \mathscr{K}$, find the degree of belief b_{ik} that f_{ik} is less or equal to all other f_{jk}, $j \in I \backslash i$.

Step 3: Transform all the relative weights of criteria w_k, $k \in \mathscr{K}$ into the degrees of belief B_k, $k \in \mathscr{K}$ that a criterion k is more or equally important as all other criteria in the set $\mathscr{K} \backslash k$; each B_k belongs to a common scale [0, 1], and is calculated using the fuzzy set comparison method described in the Appendix.

Step 4: Calculate the elements of weighted normalised $I \times K$ decision matrix $\mathscr{D} = [d_{ik}]$ with the following elements:

$d_{ik} = B_k \cdot r_{ik}$ for all columns $k \in \mathscr{K}$ which correspond to the cardinal criteria,

$d_{ik} = B_k \cdot b_{ik}$ for all columns $k \in \mathscr{K}$ which correspond to the linguistic criteria.

Step 5: Find row i*, i.e., the corresponding RP* in matrix $\mathscr{D}$ for which

$$\max_i \{\alpha \cdot \min_k d_{ik} + (1 - \alpha) \cdot \max_k d_{ik}\}$$

is achieved. The parameter α is referred to as the optimism–pessimism coefficient, it varies in the range [0, 1], the smaller the α the more optimism is expressed and vice versa.

RP* is the best-compromised replenishment policy in the sense that both its best and its worst criteria values are good enough. In the extreme two

Table 1
Inventory control variables

	Review period	Order quantity	Order-up-to-level
RP_1	13-weeks	130	—
RP_2	13-weeks	—	130
	Reorder point	Order quantity	Order-up-to-level
RP_3	10	80	—
RP_4	10	—	80

Table 2
Criteria values

	C_1	C_2	C_3
RP_1	0.979	58.85	Very high regularity
RP_2	0.987	59.93	Moderate regularity
RP_3	0.916	29.96	Moderate regularity
RP_4	0.927	32.25	Low regularity
	Very important	Moderately important	Low important

cases, for $\alpha = 0$, the max–max policy and, for $\alpha = 1$, the max–min policy is selected. These two cases express, respectively, the maximum optimism, i.e. a weight of 1 is assigned to the best criterion value and weight 0 to all others, and the maximum pessimism, where a weight of 1 is assigned to the worst criterion value and weight 0 to all others.

4. Illustrative example

Let us consider an illustrative example. The problem is to select the best among four replenishment policies RP_1, RP_2, RP_3 and RP_4, defined in Section 2.2, taking into account three optimisation criteria C_1, C_2 and C_3, given in Section 2.3. The common data, which characterise inventory environment for all RPs treated, are:

- Time unit is 1 week.
- Time horizon is 52 weeks.
- Customer demand per week is uncertain, confined to a single product and described by a linguistic expression "*about* 10 products per week", it is represented by a discretized fuzzy set: $D = \{0.25/7, 0.50/8, 0.75/9, 1/10, 0.75/11, 0.50/12, 0.25/13\}$.
- Out-of-stock policy applied is back ordering.
- Replenishment quantity is received with a fixed lead-time of 1 week.
- Unit holding cost is 10.
- Unit shortage cost is 40.
- Initial stock level is 10.

The control variables for each RP are given in Table 1. They are not optimal with respect to some criterion, but simply determined as common sense values.

Table 3
Normalized and transformed criteria values

	C_1	C_2	C_3
RP_1	0.99	0.15	0.12
RP_2	1	0.14	0.04
RP_3	0.92	0.28	0.04
RP_4	0.94	0.27	0.02

Three performance measures are defined and used: C_1 – the fill rate, C_2 – the total holding cost per item demanded, and C_3 – the regularity of replenishment orders with respect to time instant and quantities ordered. The values of criteria C_1 and C_2 are obtained by using a simulator (fuzzy inventory simulation) (FINSIM) [7]. The values of criterion C_3 are the linguistic terms defined by fuzzy sets, illustrated in Fig. 3. The relative importance of each criterion is also given by some linguistic term, see Fig. 4. Note that C_1 and C_3 are the benefit criteria and C_2 the cost criterion.

The criteria values obtained for four alternative RPs under consideration are arranged in a matrix form, given in Table 2.

After normalisation of cardinal values (Step 1 of the Algorithm) and calculation of degrees of belief (Steps 2 and 3 of the Algorithm), the weighted normalised 4×3 matrix is obtained (Step 4 of the Algorithm) and given in Table 3.

By applying the expression in Step 5 of the Algorithm the best RP* is selected. The selection depends on the optimism–pessimism coefficient α. For example, the best compromised RP*, obtained for $\alpha = 0.5$ is RP_1, which has both good fill rate and good regularity of ordering. However, RP_2 is the best for the extremely optimistic decision-makers, when $\alpha = 0$, because it has the best fill rate. In contrast, RP_3 is the best for the extremely pessimistic decision-makers, when $\alpha = 1$, since the largest of three minimum values in the three columns in Table 3, i.e., max(0.92, 0.14. 0.02) is 0.92, which corresponds to RP_3.

5. Conclusion

We conclude that multicriteria analysis is a feasible and efficient approach, and thus worth considering in choosing the best replenishment policy in the presence of uncertainty in customer demand. The algorithm developed handles, simultaneously, both cardinal values and linguistic terms, which express the performance measures of different replenishment policies. It is shown how the combined use of multicriteria concepts, description of uncertainty by means of fuzzy sets and simulation can effectively lead to identifying the best replenishment policy that maximises decision makers satisfaction with respect to more than one measure of performance.

Appendix

In this appendix a new procedure is given for calculating a measure of belief $b(\tilde{M} < \tilde{N})$ that a fuzzy relation $\tilde{M} < \tilde{N}$ between two discrete fuzzy sets $\tilde{M}$ and $\tilde{N}$ is true. For example, a fuzzy set $\tilde{M}$ and $\tilde{N}$ can be any two fuzzy sets defined in Fig. 3 or Fig. 4.

The measure $b(\tilde{M} < \tilde{N})$ is defined as the probability that a crisp value $m \in \text{support}(\tilde{M})$ is less than a crisp value $n \in \text{support}(\tilde{N})$, i.e.,

$$b(\tilde{M} < \tilde{N}) = \text{Prob}(m < n), \quad \text{(A.1)}$$

where support$(\tilde{M})$ and support$(\tilde{N})$ are the supports of the corresponding fuzzy sets. The support of a fuzzy set is defined as a crisp set that contains all the elements of the fuzzy set that have nonzero membership degrees. Let support$(\tilde{M}) = \{m_1, m_2, \dots, m_U M\}$ and support $(\tilde{N}) = \{n_1, n_2, \dots, n_U N\}$.

Let us introduce the following probabilities:

$P_{\tilde{M}}(m_l)$ – the probability that $\tilde{M}$ takes a crisp value $m_l \in \text{support}(\tilde{M})$:

$$P_{\tilde{M}}(m_l) = \mu_{\tilde{M}}(m_l)\left[\sum_{j=1}^{U^M} \mu_{\tilde{M}}(m_j)\right]^{-1}, \quad \text{(A.2)}$$

$\Phi_{\tilde{M}}(m_l)$ – the probability that $\tilde{M}$ takes crisp values which are smaller or equal to m_l.

$$\Phi_{\tilde{M}}(m_l) = \sum_{j=1}^{l} P_{\tilde{M}}(m_j). \quad \text{(A.3)}$$

Consequently, $1 - \Phi_{\tilde{M}}(m_l)$ is the probability that $\tilde{M}$ takes crisp values which are greater than m_l . Similar expressions hold for $\tilde{N}$.

Then, under assumptions that random values defined on support$(\tilde{M})$ and support$(\tilde{N})$ are independent, (A.1) becomes

$$\begin{aligned} b(\tilde{M} < \tilde{N}) &= \sum_{l=1}^{U^M} \text{Prob}[m = m_l \wedge n > m_l] \\ &= \sum_{l=1}^{U^M} P_{\tilde{M}}(m_l)[1 - \Phi_{\tilde{N}}(m_l)]. \quad \text{(A.4)} \end{aligned}$$

In addition, from probability theory and fuzzy sets theory, the following relationships related to the measure $b(\tilde{M} < \tilde{N})$ hold:

$$b(\tilde{M} < \tilde{N}) = 1 - b(\tilde{M} \geqslant \tilde{N}), \quad \text{(A.5)}$$

$$\begin{aligned} &b(\tilde{M} < \{\tilde{N}_1 \wedge \tilde{N}_2 \wedge \tilde{N}_3 \wedge \cdots)\} \\ &= \min\{b(M < \tilde{N}_1), b(M < \tilde{N}_2), b(M < \tilde{N}_3), \dots\}, \quad \text{(A.6)} \end{aligned}$$

where $\tilde{N}_1, \tilde{N}_2, \tilde{N}_3, \cdots$ are fuzzy sets.

References

[1] B. Kosko, Fuzzy Thinking, the New Science of Fuzzy Logic, Flamingo, London, 1994.
[2] R.H. Wilson, A scientific routine for stock control, Harvard Business Review 13 (1934) 116–128.

[3] A.F. Veinott, J.R. Wagner, H.M. Wagner, Computing original (s, S) inventory policies, Management Science 11 (1965) 525–552.
[4] E.L. Porteus, Numerical comparisons of inventory policies for periodic review systems, Operations Research 33 (1985) 132–152.
[5] A. Chikan (Ed.), Bibliography of Inventory Literature, Vol. III, Mathematical Models of Inventories, ISIR, Budapest, 1994.
[6] M. Vujosevic, D. Petrovic, R. Petrovic, EOQ formula when inventory cost is fuzzy, International Journal Production Economics, 45 (1996) 499–504.
[7] D. Petrovic, Modelling and analysis of serial supply chains in uncertain environments, PhD Thesis, University of Warwick, UK, 1997.
[8] J.D. Hey, Uncertainty in Micro-economics, Martin Robertson, Oxford, 1979.