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An optimal batch size for a JIT manufacturing system

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Abstract

This paper addresses the problem of a manufacturing system that procures raw materials from suppliers in a lot and processes them to convert to finished products. It proposes an ordering policy for raw materials to meet the requirements of a production facility. In turn, this facility must deliver finished products demanded by outside buyers at fixed interval points in time. In this paper, first we estimate production batch sizes for a JIT delivery system and then we incorporate a JIT raw material supply system. A simple algorithm is developed to compute the batch sizes for both manufacturing and raw material purchasing policies. Computational experiences of the problem are also briefly discussed.

Keywords: Inventory control; Materials procurement; JIT; Periodic delivery; Optimum order quantity

1. Introduction

A desirable condition in long-term purchase agreements in a just-in-time (JIT) manufacturing environment is the frequent delivery of small quantities of items by suppliers/vendors so as to minimize inventory holding costs for the buyer. Consider a manufacturing system that procures raw materials from outside suppliers and processes them to convert into finished products for retailers/customers. The manufacturer must deliver the products in small quantities to minimize the retailer's holding cost, and accept the supply of small quantities of raw materials to minimize his own holding costs. In the traditional JIT environment, the supplier of raw materials is dedicated to the manufacturing firm, and normally is located close by. The manufacturing lot size is dependent on the retailer's sales volume (/market demand), unit product cost, set-up cost, inventory holding cost and transportation cost. The raw material purchasing lot size is dependent on raw material requirement in the manufacturing system, unit raw material cost, ordering cost and inventory holding cost. Therefore, the optimal raw material purchasing

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Nomenclature

D_p	demand rate of a product p, units per year
P	production rate, units per year (here $P > D_p$)
Q_p	production lot size
H_p	annual inventory holding cost, \$/unit/year
A_p	set-up cost for a product p (\$/set-up)
f_r	amount/quantity of raw material r required in producing one unit of a product
D_r	demand of raw material r for the product p in a year, $D_r = f_r D_p$
Q_r	ordering quantity of raw material r
A_r	ordering cost of a raw material r
H_r	annual inventory holding cost for raw material r
PR_r	price of raw material r
Q_r^*	optimum ordering quantity of raw material r
x	shipment quantity to customer at a regular interval (units/shipment)
L	time between successive shipments $= x/D_p$
T	cycle time measured in year $= Q_p/D_p$
m	number of shipments during the cycle time $= T/L$
n	number of shipments during production uptime
T_1	production uptime in a cycle
T_2	production downtime in a cycle $= T - T_1$
$\mathrm{IP}_{\mathrm{avg}}$	average finished goods inventory

quantity may not be equal to the raw material requirement for an optimal manufacturing batch size. To operate the JIT manufacturing system optimally, it is necessary to optimize the activities of both raw material purchasing and production lot sizing simultaneously, taking all the operating parameters into consideration. Unfortunately, until recently, most JIT studies in the literature have been descriptive (Aderohunmu, Mobolurin, & Bryson, 1995; Chapman & Carter, 1990), and most of the analytical studies do not take all the costs for both sub-systems into consideration (Ansari & Hechel, 1987; Ansari & Modarress, 1987; O'Neal, 1989). Therefore, the overall result may not be optimal.

A larger manufacturing batch size reduces the set-up cost component to the overall unit product cost. The products produced in one batch (/one manufacturing cycle) are delivered to the retailer in m small lots (ie m retailer cycles per one manufacturing cycle) at fixed time intervals. So the inventory forms a saw tooth pattern during the production uptime and a staircase pattern during production downtime in each manufacturing cycle. Likewise, the manufacturer receives n small lots of raw material, at regular intervals, during the production uptime of each manufacturing cycle. The raw materials are consumed at a given rate during the production uptime only. It is assumed that the production rate is greater than the demand rate. So the accumulated inventory during production uptime is used for making delivery during production downtime until the inventory is exhausted (see Fig. 1). Production is then resumed and the cycle repeated.

Sarker, Karim and Azad (1993, 1995a) and Sarker, Karim and Haque (1995b) developed a model operating under continuous supply at a constant rate. In their paper, they considered two cases. In case I, the ordering quantity of raw material is assumed to be equal to the raw material required for one batch of the production system. The raw material, which is replenished at the beginning of a production cycle, will

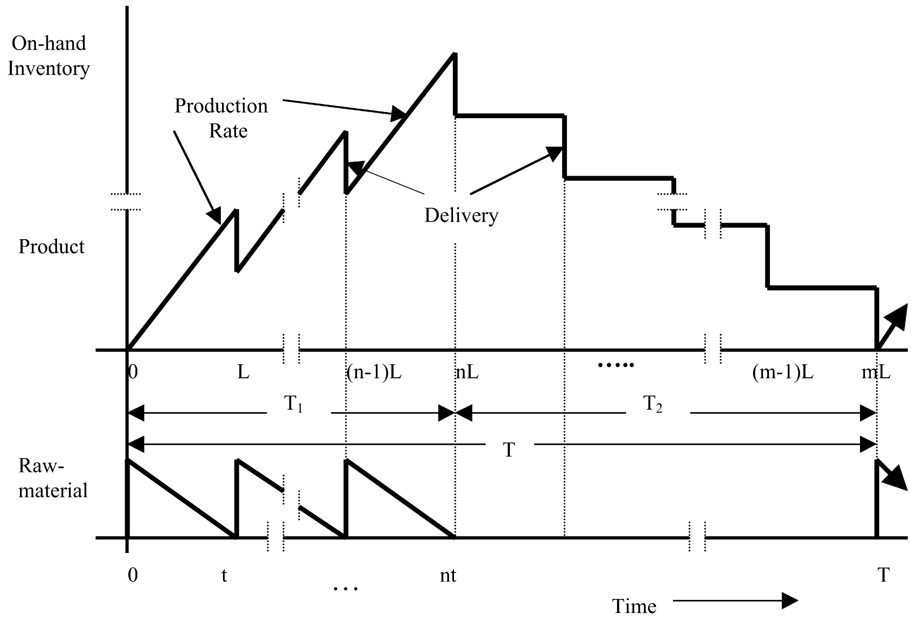

Fig. 1. Inventory level with time for product (top), and raw material (bottom).

be fully consumed at the end of this production run. In case II, it is assumed that the ordering quantity of a raw material to be n times the quantity required for one lot of a product, where n is an integer. Though case I can be fitted in the JIT supply system, case II is not favourable for the JIT environment.

An ordering policy, for raw materials to meet the requirements of a production facility under a fixed quantity, periodic delivery policy, has been developed by Golhar and Sarker (1992), Jamal and Sarker (1993), Sarker and Golhar (1993), and Sarker and Parija (1994). They considered that the manufacturer is allowed to place only one order for raw materials per cycle, which is similar to case I above. In this case, a fixed quantity of finished goods (say x units) is to be delivered to the customer at the end of every L units of time (fixed interval). This delivery pattern forces inventory build-up in a saw-tooth fashion during the production uptime. The on-hand inventory depletes sharply at a regular interval during the production downtime until the end of the cycle time, forming a staircase pattern. Recently, Sarker and Parija (1996) developed another model for purchasing a big lot of raw material that will be used in n consecutive production batches.

In this paper, our purpose is to incorporate JIT concept in conventional joint batch sizing problems in two stages. In the first stage, the problem considers a JIT delivery system. This problem is somewhat similar to Sarker and Parija (1994), which considers a fixed quantity-periodic delivery policy. The raw material purchasing quantity is equal to the exact requirement of one manufacturing cycle. Chakravarty and Martin (1991) and Golhar and Sarker (1992) perceived that the production time might not be composed of exactly m full shipment periods. To satisfy the demand for the whole cycle time, the production run length may extend for a fraction of a shipment period in addition to m shipments. However, when the transportation cost is independent of the shipment quantity, the fractional shipment may not be cost effective. The retailer may not accept a fractional shipment either. So we consider the frequency of the finished product supply, m, strictly as an integer. In the second stage, the problem considers a JIT system for both finished product delivery and raw material receiving. This problem is similar to the first problem except that the raw material requirements in one manufacturing cycle will be purchased in n small lots. Each lot of raw material will be consumed in L units of time during the production uptime. The motivation of this research comes from the intention of examining the benefits of JIT systems when incorporated in joint batch sizing policy because:

- JIT systems are successful for many practical inventory systems, and
- joint inventory policy is considered to be an effective policy for multi-stage inventory systems.

The cost factors considered are ordering/setting up, holding and transportation costs. We have devised the total cost equation of the system with respect to the production quantity, finished product delivery frequency and/or raw material supply frequency and then solved the problem as an unconstrained optimization problem.

The paper is organized as follows. Following the introduction, the mathematical formulations of two coordinated policies are given. Then, the solution methodology is provided, followed by results and discussions. Finally, conclusions are drawn.

2. Model formulation

The total quantity manufactured during the production time, T_1, must exactly match the demand for cycle time, T. It is assumed that T_1 must be less than or equal to T. In order to ensure no shortage of products, we assume that the production rate, P, is greater than the demand rate of the finished products, D_p. There is a demand of x units of finished goods at the end of every L time units (due to fixed-interval batch supply) and the quantity produced during each period (of L time units) is PL, where $(PL - x)$ is positive. So the finished goods inventory build-up forms a saw-tooth pattern during the production period (see Fig. 1). The on-hand inventory depletes sharply at a regular interval after the production run till the end of the cycle time. The later one forms a staircase pattern.

In this paper, we consider two cases of JIT manufacturing environment. Case I: JIT delivery system, where the delivery of finished goods is in m small lots of x units at regular time intervals and the raw material purchasing quantity is exactly equal to the requirement of raw material in a manufacturing cycle. Case II: JIT supply and delivery system, modifies the raw material supply pattern of case I. In this case, the raw material required in one manufacturing cycle will be received in n small lots at regular time intervals during the production uptime. For simplicity, we assume that the length of time interval L is equal for both finished goods delivery and raw material receiving. However, the carrying of raw material is not permitted during production downtime in any of the cases. If $m = 1$, the cases revert to the classic inventory models, where all the finished goods are delivered in one lot in a cycle and it does not have the JIT element any more. For a very large m, they become a continuous delivery system.

2.1. Assumptions

To simplify the analysis, we make the following assumptions:

- There is only one manufacturer and only one raw material supplier for each item.
- The production rate is uniform and finite.

- There are no shortages.
- The delivery of the product is in a fixed quantity at a regular interval.
- The raw material supply is available in a fixed quantity whenever required.
- The producer is responsible to transport the product to the retailers' location.

2.2. Case I: JIT delivery system

The total cost function, for case I, can be expressed as follows:

$$\mathrm{TC}_1 = \frac{D_r}{Q_r}A_r + \frac{D_p}{P}\left(\frac{Q_r}{2}\right)H_r + \frac{D_p}{Q_p}A_p + \mathrm{IP}_{\mathrm{avg}}H_p \quad (1)$$

where

$$\frac{D_r}{Q_r}A_r = \text{ordering cost of raw materials}$$

$$\frac{D_p}{P}\left(\frac{Q_r}{2}\right)H_r = \text{inventory holding cost of raw materials}$$

$$\frac{D_p}{Q_p}A_p = \text{set-up cost of finished products}$$

$$\mathrm{IP}_{\mathrm{avg}}H_p = \text{inventory holding cost of finished products}$$

The first two terms in Eq. (1) represent the inventory cost for raw material and the next two terms represent the inventory costs for finished goods. Since the raw materials are carried for only T_1 time units, the holding cost in second term, H_r, is rescaled by the factor $T_1/T = D_p/P$ to compute the raw material inventory carrying cost. Also, unlike the assumption of a one-to-one conversion of raw materials to finished goods a raw material may be transformed to a new product through the manufacturing process at a different conversion rate, $f_r = D_r/D_p = Q_r/Q_p$. Therefore, substituting D_p/Q_p for D_r/Q_r, and f_rQ_p for Q_r, Eq. (1) leads to

$$\mathrm{TC}_1 = \frac{D_p}{Q_p}A_r + \frac{D_p}{P}\left(\frac{f_rQ_p}{2}\right)H_r + \frac{D_p}{Q_p}A_p + \mathrm{IP}_{\mathrm{avg}}H_p \quad (2)$$

in which the total inventory costs for both raw materials and finished goods are expressed in terms of manufacturing batch size, Q_p. The average finished goods inventory per cycle can be expressed as follows:

$$\mathrm{IP}_{\mathrm{avg}} = Q_p\left(1 - \frac{D_p}{2P}\right) - \left(\frac{m-1}{2}\right)x \quad (3)$$

The expression for $\mathrm{IP}_{\mathrm{avg}}$ is obtainable by calculating the relevant area of Fig. 1 and dividing it by mL. The area consists of n triangles of area $PL^2/2$, $(n^2 - n)/2$ rectangles of area $(PL^2 - Lx)$, and a number of rectangles of area Lx.

The details of the derivation of this expression can be found for integer m in Sarker and Uddin (1995) and for continuous m in Sarker and Parija (1994). For simplicity, m has been assumed to be an integer in our study.

Using the relationship in Eq. (3), the total cost in Eq. (1) may be written as

$$\mathrm{TC}_1 = \frac{D_p}{Q_p}(A_r + A_p) + \frac{D_p}{P}\left(\frac{f_r Q_p}{2}\right)H_r + Q_p\left(1 - \frac{D_p}{2P}\right)H_p - \left(\frac{m-1}{2}\right)xH_p \quad (4)$$

After simplification, Eq. (4) becomes

$$\mathrm{TC}_1 = \frac{D_p}{Q_p}(A_r + A_p) + Q_p\left(\frac{D_p}{2P}f_r H_r - \frac{D_p}{2P}H_p + H_p\right) - \frac{xH_p}{2}m + \frac{xH_p}{2} \quad (5)$$

It is required to minimize the function TC_1 to determine Q_p where $Q_p = mx$.

2.3. Case II: JIT Supply and delivery system

The total cost function, for case II, can be expressed as follows:

$$\mathrm{TC}_2 = \frac{D_p}{Q_p}A_p + Q_p\left(1 - \frac{D_p}{2P}\right)H_p + \frac{D_r}{Q_r}A_r + \frac{D_p}{P}\left(\frac{f_r Q_r}{2}\right)H_r - \left(\frac{m-1}{2}\right)xH_p \quad (6)$$

where

$\frac{D_p}{Q_p}A_p$ = set-up cost of finished products

$Q_p\left(1 - \frac{D_p}{2P}\right)H_p - \left(\frac{m-1}{2}\right)xH_p$ = inventory holding cost of finished products

$\frac{D_r}{Q_r}A_r$ = ordering cost of raw materials

$\frac{D_p}{P}\left(\frac{f_r Q_r}{2}\right)H_r$ = inventory holding cost of raw materials

It is required to minimize the function TC_2 to determine Q_p where $Q_p = mx$, $Q_p = nQ_r$ and $Q_r = PL$. Here $Q_p = mx$ and $Q_p = nQ_r$ indicate that mx must be equal to nQ_r. In other words, Q_r is equal to the ratio of m and n multiplied by x that is $(m/n)x$. In this paper, we assume that the time intervals for raw material receiving will be L time units. So we can write $Q_r = PL$, that means the raw material purchasing quantity is known and it is independent of Q_p. Substituting the relationship $Q_r = PL$ in Eq. (6) and then simplifying, we get the total cost equation as follows:

$$\mathrm{TC}_2 = \frac{D_p}{Q_p}A_p + Q_p\left(1 - \frac{D_p}{2P}\right)H_p - \frac{xH_p}{2}m + \frac{f_r D_p}{PL}A_r + \frac{D_p}{P}\left(\frac{f_r PL}{2}\right)H_r + \frac{xH_p}{2} \quad (7)$$

3. Solution methodology

In this section, the solution procedures for both the problems stated in Section 2 are presented.

Efficient algorithms may be applied to solve the above problems using an optimization technique. To solve a similar problem, Moinzadeh and Aggarwal (1990) proposed an algorithm for discrete optimization, Park and Yun (1984) developed a heuristic method, and Golhar and Sarker (1992) used one-directional search procedure. In this paper, we propose a heuristic method which is simple, easy to apply and computationally efficient. In this method, an initial solution for Q_p is determined by relaxing the integrality requirement for m. The corresponding value of m is computed. If the computed m is not integer, then we search for an integer m in the neighbouring points, which gives minimum TC.

3.1. Problem I

In order to find an optimal manufacturing quantity, it is necessary to differentiate TC_1 with respect to Q_p (Eq. (5)). But the function is non-differentiable as it contains an integer variable m. However, it can be shown that TC is a convex function of Q_p for a given m. As we assumed in an earlier section, the integer variable m can be replaced by Q_p/x. So the new cost equation is

$$\mathrm{TC}_1 = \frac{D_p}{Q_p}(A_r + A_p) + Q_p\left(\frac{D_p}{2P}f_r H_r - \frac{D_p}{2P}H_p + \frac{H_p}{2}\right) + \frac{xH_p}{2} \tag{8}$$

It can be shown that TC_1 is a convex function. Now, differentiating TC_1 with respect to Q_p and then equating to zero, we get

$$Q_p^* = \sqrt{\frac{D_p(A_p + A_r)}{\mathrm{KK}}} \tag{9}$$

where

$$\mathrm{KK} = \frac{D_p}{2P}f_r H_r - \frac{D_p}{2P}H_p + \frac{H_p}{2}$$

Now the calculated m $(= Q_p^*/x)$ may not be an integer as we replaced m by a continuous variable in Eq. (8). In the case of non-integer m, we take the neighboring integer value corresponding to which the total cost is minimum.

The algorithm to solve the batch sizing problem for problem I is as follows:

Algorithm I: finding batch size

Step 0. Initialize and store D_p, P, A_p, A_r, H_p and H_r.
Step 1. Compute the number of batch size Q_p^* using Eq. (9).
Step 2. Compute m $(= Q_p^*/x)$. If m is an integer, then stop.
Step 3. Compute TC_1 using Eq. (5) for $m = \lfloor m \rfloor$ and $\lceil m \rceil$.
Choose the $m^* = m$ that gives minimum TC_1.
Stop.

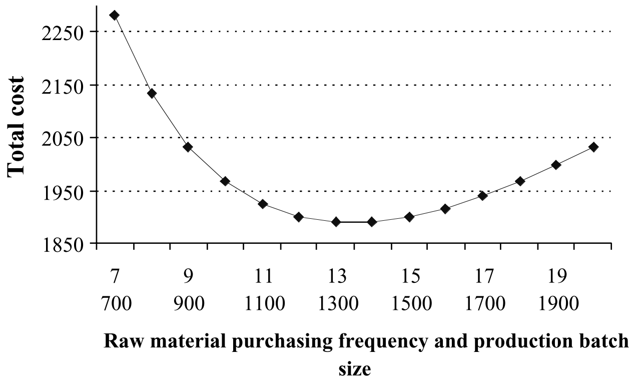

Fig. 2. Total cost curve.

3.2. Problem II

Substituting the integer variable m by Q_p/x, we can rewrite Eq. (7) as follows:

$$TC_2 = \frac{D_p}{Q_p}A_p + Q_p\left(1 - \frac{D_p}{2P}\right)H_p - \frac{H_p}{2}Q_p + \frac{D_r}{PL}A_r + \frac{D_p}{P}\left(\frac{f_rPL}{2}\right)H_r + \frac{xH_p}{2} \tag{10}$$

It can also be shown that TC_2 is a convex function. Now, differentiating TC_2 with respect to Q_p and then equating to zero, we get

$$Q_p^* = \sqrt{\frac{D_pA_p}{\frac{H_p}{2}\left(1 - \frac{D_p}{P}\right)}} \tag{11}$$

Now the calculated m (= Q_p^*/x) may not be an integer as we replaced m by a continuous variable in Eq. (10). In case of non-integer m, we take the neighbouring integer value corresponding to which the total cost is minimum.

The algorithm to solve the batch sizing problem for problem II is as follows:

Algorithm II: finding batch size

Step 0. Initialize and store D_p, P, A_p, A_r, H_p and H_r.
Step 1. Compute the number of batch size Q_p^* using Eq. (11).
Step 2. Compute m (= Q_p^*/x). If m is an integer, then stop.
Step 3. Compute TC_2 using Eq. (7) for $m = \lfloor m \rfloor$ and $\lceil m \rceil$. Choose the $m^* = m$ that gives minimum TC_2. Stop.

4. Results and discussions

The total cost functions with integer m are plotted in Fig. 2. As an assumption made $m = Q_p/x$ in an earlier section, the total cost is computed for each paired m (integer) and Q_p data. The data used, to

Table 1
Optimum TC, m and Q_p (* indicates optimum TC)

Calculated values		With upper integer (m)		With lower Integer (m)		Solution	
Q_p	m	m	TC	m	TC	m^*	Q_p^*
1342	13.42	14	1890.476	13	1889.744*	13	1300

generate these curves, are similar to those in Jamal and Sarker (1993) and Sarker and Parija (1994): $D_p = 2400$ units/year, $P = 3600$ units/year, $A_p = \$300$/set-up, $A_r = \$200$/order, $H_p = \$2$/unit/year, $H_r = \$1$/unit/year, $f_r = 1$ and $x = 100$ units/shipment. The behavior of the cost function, in Fig. 2, reflects a smooth and convex pattern.

If we restrict m to an integer for the problems stated in Golhar and Sarker (1992), Jamal and Sarker (1993), and Sarker and Parija (1994), then their cost equations become similar to our Eq. (5). With the data mentioned earlier, the Optimum TC, m and Q_p are shown in Table 1. The solution is $Q_p^* = 1300$, $m^* = 13$ and TC = \$1889.744. As per the solutions of Golhar and Sarker (1992), $Q_p =$ 1342 units/batch which is incidentally similar to our Q_p solution with Eq. (9), shown in the first column of Table 1. The optimum Q_p is 1346 as determined by Jamal and Sarker (1993) and Sarker and Parija (1994).

The experimental analysis of the total cost against raw material ordering cost for both case 1 and case 2 shows that case 2 is more sensitive than case 1 to the ordering cost when other parameters are fixed. From the analysis, it can be concluded that case 2 is more attractive for very low ordering cost. That means if the ordering cost is very low, which is a condition for JIT supply, case 2 can achieve more savings, which is consistent with the reasons for using JIT systems.

5. Conclusions

In a JIT manufacturing environment, a supplier is expected to deliver goods frequently in small lots. Ideally, a supplier to the JIT buyer is expected to synchronize his production capacity with the buyer's demand so that the inventory in the supply pipeline is reduced and eventually eliminated.

An inventory model is developed for perfect matching. The total cost function is non-differentiable with respect to manufacturing batch size. However, it can be shown that the total cost function is convex for a given m. A simple algorithm is developed to compute optimal manufacturing batch size and the associated raw material purchasing lot size. The quality of solution is discussed and sensitivity analysis is provided.

In problem II, it is assumed that $Q_p = nQ_r$, where $Q_r = PL$. To generalize the problem, the relation $Q_r = PL$ needs to be relaxed. That is a topic for further research.

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