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Multiple-attribute decision making methods for plant layout design problem

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Abstract

The layout design problem is a strategic issue and has a significant impact on the efficiency of a manufacturing system. Much of the existing layout design literature that uses a surrogate function for flow distance or for simplified objectives may be entrapped into local optimum; and subsequently lead to a poor layout design due to the multiple-attribute decision making (MADM) nature of a layout design decision. The present study explores the use of MADM approaches in solving a layout design problem. The proposed methodology is illustrated through a practical application from an IC packaging company. Two methods are proposed in solving the case study problem: Technique for order preference by similarity to ideal solution (TOPSIS) and fuzzy TOPSIS. Empirical results showed that the proposed methods are viable approaches in solving a layout design problem. TOPSIS is a viable approach for the case study problem and is suitable for precise value performance ratings. When the performance ratings are vague and imprecise, the fuzzy TOPSIS is a preferred solution method.

Keywords: Plant layout; Fuzzy set theory; Multiple-attribute decision making; TOPSIS

1. Introduction

Layout design invariably has a significant impact on the performance of a manufacturing or service industry system, and consequently has been an active research area for several decades [1,2]. Much of the plant layout design literature is either algorithmic or of a procedural type. The former approach, such as Spiral® [3] and MULTIPLE [4], can efficiently generate alternative layout designs, but the design objectives are often over-simplified. For example, the resulting departmental shapes often deviate from practical constraints. For another example, the flow distance, either measured in Euclidean or rectilinear distance, and so may not represent the physical flow distance. This is particularly important when there are qualitative design criteria; causing the resulting layout design to lack functionality and credence for a quality solution.

Additional algorithmic approaches could model the layout design problem as a mixed integer programming formulation [5–8]. These approaches use the flow distance as the surrogate function, and are often computationally prohibitive.

The procedural approach, such as the systematic layout planning procedure [9], has the flexibility to incorporate a variety of design objectives but is often lacking sound theoretical foundation and credence to be a quality solution [10].

The layout decision is usually based on both quantitative and qualitative performance ratings pertaining to the desired *closeness* or *closeness relationships* among the facilities. The 'closeness' is a vague notion that captures issues such as the material flow and the ease of employee supervision [11]. Clearly, the evaluation of critical criteria for a layout design is often a challenging and complex task [12,13].

The present study focuses on the evaluation of alternative layout designs by considering both qualitative and quantitative design criteria. It simultaneously evaluates all

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the desired criteria for design alternatives. This will permit the desired design criteria to be better incorporated and evaluated. In addition, the direct evaluation of a design alternative in lieu of incomplete design, e.g., an improvement type layout design algorithm, will increase the level of confidence in searching for a quality solution. It solves a layout design problem using multiple-attribute decision making (MADM) methods. It seeks to evaluate a large number of layout design alternatives generated by an efficient layout design algorithm. The evaluation of a large number of design alternatives will thereby reduce the risk of missing a high-quality solution.

We propose two MADM methods in solving a plant layout design problem. They are: technique for order preference by similarity to ideal solution (TOPSIS) and fuzzy TOPSIS. A case study from an integrated-circuit (IC) packaging plant is adopted for the empirical testing.

The remainder of this paper is organized as follows. The pertinent literature is reviewed in Section 2. Section 3 provides the background information for the case study problem. The theories and empirical description for the two methods are discussed in detail sequentially in Sections 4 and 5. The discussion that summarizes the empirical results is given in Section 6. Finally, conclusions and future research opportunities are drawn together in Section 7.

2. Literature review

Karray et al. [11] proposed an integrated methodology using the fuzzy set theory and genetic algorithms to investigate the layout of temporary facilities in relation to the planned buildings in a construction site. It identifies the closeness relationship values between each pair of facilities in a construction site using fuzzy linguistic representation.

Grobelny [14,15] explored the use of a fuzzy approach to facilities layout problems using a fuzzy criterion to determine the closeness relationship among departments; and then to determine the final optimum design. Evans et al. [16] and Dweiri and Meier [17] used a similar concept that employed the theory of fuzzy sets to solve a block layout design problem.

Raoot and Rakshit [18] proposed a construction-type layout design heuristic based on fuzzy set theory. A linguistic variable was used to model various qualitative design criteria, and then to determine the closeness relationship among departments. The resulting closeness relationship matrix was used to construct a layout design. This approach allowed, in a qualitative manner, for the systematic treatment of uncertainty due to fuzziness.

All of the above fuzzy-based layout design algorithms modeled the fuzzy or linguistic closeness relationship among departments. The resulting fuzzy scores that represent the desired closeness are then used for a layout design criterion along as part of the layout improvement process. In these methods, the fuzzy closeness determines the order of entry of departments into the layout; but the department placement and departmental dimensions are not explicitly considered.

Badiru and Arif [19] proposed a fuzzy linguistic expert system in solving a layout design problem. It incorporated an existing layout algorithm, BLOCPLAN [20], to efficiently create design alternatives. Their proposed expert system is an integrated system with three major components—fuzzy algorithm, BLOCPLAN and expert system (knowledge-based rules). The interactions among the three components have the merits of computational efficiency and fuzzy linguistic modeling capability for a layout design problem. The system is fundamentally an improvement-type layout design algorithm.

In the study of decision making, terms such as multiple objective, multiple attribute and multiple criteria are often used interchangeably. Here, we provide the conceptual distinctions leading to the definition of the proposed MADM methods. However, more detailed information can be found in Ribeiro [21–23].

Multiple objective decision making (MODM) consists of a set of conflicting goals that cannot be achieved simultaneously. It invariably concentrates on continuous decision spaces and can be solved with mathematical programming techniques. MODM generally deals with (i) preferences relating to the decision maker's objectives and (ii) the relationships between objectives and attributes. An alternative could be described whether in terms of its attributes or in terms of the attainment of the decision maker's objectives.

MADM deals with the problem of choosing an option from a set of alternatives which are characterized in terms of their attributes. MADM is a qualitative approach due to the existence of criteria subjectivity. It requires information on the preferences among the instances of an attribute, and the preferences across the existing attributes. The decision maker may express or define a ranking for the attributes as importance/weights. The aim of the MADM is to obtain the optimum alternative that has the highest degree of satisfaction for all of the relevant attributes.

TOPSIS and fuzzy TOPSIS have been applied to solve a variety of applications, and are proven methodology in solving MADM problems [24,25]. The present study explores the use of TOPSIS and fuzzy TOPSIS to solve the proposed layout design problem since we are not aware of a similar application. Details of the proposed case and methodology are discussed sequentially in the following sections.

3. The case

The layout design problem presented in Yang and Kuo [10] is adopted for the present study. It is an IC packaging plant. The detail of IC fabrication process is not discussed in this paper for a concise presentation. Interested readers are referred to Xiao [26] for a detailed discussion of the IC fabrication process.

The IC packaging plant usually adopts the process layout strategy that clusters the same tool type to form a workstation. A product traverses all the workstations in the same sequence. For the case study problem, there are ten departments (workstations) whose names and area size information are depicted in Table 1.

The case study is based on an existing layout design. Understandably, the company would like to know whether the existing design is an effective one. The experience learned from this study will provide guidelines for the company's future layout design problem, as well as for identifying potential layout improvement opportunities.

In Yang and Kuo [10], a set of potential 'good' layout alternatives were generated by commercial software, entitled Spiral® [3]. According to the flow distance criterion, the top 17 layout design alternatives were generated and selected for further analysis. The existing layout design was the 18th alternative choice. A preliminary study was conducted to determine the design criteria among the area experts that subsequently led to three quantitative and three qualitative design attributes. The quantitative attributes included *material handling distance* (in 'meters'), *adjacency score* and *shape ratio*, which are the direct outputs of Spiral®. They are referred to as C_1, C_2 and C_3, respectively, hereafter.

Table 1
Layout data

No.	Department name	Size (m^2)
1	Wafer sawing	89.21
2	Die bond	181.51
3	Wire bond	577.38
4	Molding	599.57
5	Dejunk/trimming & curing	183.71
6	Electro deflash/solder platting	500.13
7	Marking	199.94
8	Forming and singulation	186.40
9	Lead scanning/inspection	110.78
10	Packaging	51.09

The handling distance was measured by the sum of the products of flow volume and rectilinear distance between the centroids of two departments. The adjacency score is the sum of all positive relationships between adjacent departments. There is a positive relationship between each two consecutive departments along the process routing. Shape ratio is defined as the maximum of the depth-to-width and width-to-depth ratio of the smallest rectangle that completely encloses the department. The shape ratio is always greater or equal to one. For a layout design problem, we endeavour to minimize both the shape ratio and flow distance, while maximizing adjacency score.

There are three qualitative attributes—*flexibility*, *accessibility* and *maintenance*. They are referred to as C_4, C_5 and C_6, respectively, hereafter. Flexibility involves two aspects: the first is the capability to perform a variety of tasks under a variety of operating conditions; second is the flexibility of future expansion. Accessibility involves material handing and operator paths. Finally, the maintenance issue involves the required space for maintenance engineers and tool movement. The qualitative attributes are evaluated using an analytic hierarchy process (AHP). Readers are referred to Yang and Kuo [10] for details of the AHP evaluation process, as well as the block layout figures of the 17 design alternatives. The performance ratings for the 18 alternatives with respect to the six attributes are summarized in Table 2: decision matrix.

Yang and Kuo [10] adopted a data envelopment analysis (DEA) approach in solving the case study problem. DEA is

Table 2
Decision matrix

No.	C_1	C_2	C_3	C_4	C_5	C_6
A_1	185.9500	8.0000	8.2800	0.0119	0.0260	0.0690
A_2	207.3700	9.0000	3.7500	0.0595	0.0260	0.0575
A_3	206.3800	8.0000	7.8500	0.0714	0.0519	0.0345
A_4	189.6600	8.0000	8.2800	0.0714	0.0779	0.0460
A_5	211.4600	8.0000	7.7100	0.0714	0.0390	0.0460
A_6	264.0700	5.0000	2.0700	0.0357	0.0519	0.0690
A_7	228.0000	8.0000	14.0000	0.0476	0.0390	0.0230
A_8	185.5900	9.0000	6.2500	0.0476	0.0130	0.0575
A_9	185.8500	9.0000	7.8500	0.0357	0.0260	0.0575
A_{10}	236.1500	8.0000	7.8500	0.0595	0.0779	0.0690
A_{11}	183.1800	8.0000	2.0000	0.0952	0.1169	0.0920
A_{12}	204.1800	8.0000	13.3000	0.0357	0.0390	0.0575
A_{13}	225.2600	8.0000	8.1400	0.0714	0.0390	0.0345
A_{14}	202.8200	8.0000	8.0000	0.0357	0.0779	0.0575
A_{15}	170.1400	9.0000	8.2800	0.0952	0.1169	0.0920
A_{16}	216.3800	9.0000	7.7100	0.0476	0.0519	0.0690
A_{17}	179.8000	8.0000	10.3000	0.0476	0.0779	0.0345
A_{18}	185.7500	10.0000	10.1600	0.0595	0.0519	0.0345

a nonparametric approach that does not require any assumptions about the functional form of the production function. Assume that there are n decision-making units (DMUs) to be evaluated. Each DMU consumes varying amount of m different inputs to produce s different outputs. Let

DMU_k = the kth decision making unit (DMU), $k = 1, 2, \ldots, n$;

X_{ik} = the ith input for the kth DMU, $i = 1, 2, \ldots, m$ and $k = 1, 2, \ldots, n$;

Y_{rk} = the rth output for the kth DMU, $r = 1, 2, \ldots, s$ and $k = 1, 2, \ldots, n$;

v_i = the associated weight for the ith input, $i = 1, 2, \ldots, m$;

u_r = the associated weight for the rth output, $r = 1, 2, \ldots, s$; and

h_k = efficiency score ($h_k \leqslant 1$).

Specifically, DMU_k consumes amount X_{ik} of input i and produces amount Y_{rk} of output r, that can be incorporated into an efficiency measure—the weighted sum of the outputs divided by the weighted sum of the inputs $h_k = \sum u_r Y_{rk} / \sum v_i X_{ik}$. This definition requires a set of factor weights u_r and v_i, which are the decision variables. Each DMU_k is assigned the highest possible efficiency score ($h_k \leqslant 1$) by choosing the optimal weights for the outputs and inputs [27]. DEA has been applied to a variety of applications for choosing performance frontiers [28].

There are constraints for the application of the DEA in solving a layout design problem. First, it requires at least two design alternatives or decision-making units for each input or output measure [29]. Thus, it may become a constraint when there are many performance measures. Second, the idea of performance frontiers often generates several popular choices that all lie along the DEA frontier line; it is difficult to realize the discrepancy among those top choices.

This research explores the use of TOPSIS and fuzzy TOPSIS in solving the proposed layout design problem. The TOPSIS uses specific values for MADM problem, while the fuzzy TOPSIS is applied to the instances of imprecise and fuzzy performance ratings.

4. TOPSIS

4.1. Principles of TOPSIS

A MADM problem can be concisely expressed in a matrix format, in which columns indicate attributes considered in a given problem; and in which rows list the competing alternatives. Specifically, a MADM problem with m alternatives ($A_1, A_2, \ldots, A_m$) that are evaluated by n attributes ($C_1, C_2, \ldots, C_n$) can be viewed as a geometric system with m points in n-dimensional space. An element x_{ij} of the matrix indicates the performance rating of the ith alternative, A_i, with respect to the jth attribute, C_j, as shown in Eq. (1):

$$D = \begin{array}{c} \\ A_1 \\ A_2 \\ A_3 \\ \vdots \\ A_m \end{array} \begin{array}{c} \begin{array}{ccccc} C_1 & C_2 & C_3 & \cdots & C_n \end{array} \\ \begin{bmatrix} x_{11} & x_{12} & x_{13} & \cdots & x_{1n} \\ x_{21} & x_{22} & x_{23} & \cdots & x_{2n} \\ x_{31} & x_{32} & x_{33} & \cdots & x_{3n} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ x_{m1} & x_{m2} & x_{m3} & \cdots & x_{mn} \end{bmatrix} \end{array}. \quad (1)$$

Hwang and Yoon [30] developed TOPSIS based on the concept that the chosen alternative should have the shortest distance from the positive ideal solution and the longest distance from the negative ideal solution. More detailed information can be found in Yoon and Hwang [24]. The terms used in the present study are briefly defined as follows:

- *Attributes*: Attributes ($C_j, j = 1, 2, \ldots, n$) should provide a means of evaluating the levels of an objective. Each alternative can be characterized by a number of attributes.
- *Alternatives*: These are synonymous with ‘options’ or ‘candidates’. Alternatives ($A_i, i = 1, \ldots, m$) are mutually exclusive of each other.
- *Attribute weights*: Weight values (w_j) represent the relative importance of each attribute to the others. $W = \{w_j | j = 1, 2, \ldots, n\}$.
- *Normalization*: Normalization seeks to obtain comparable scales, which allows attribute comparison. The vector normalization approach divides the rating of each attribute by its norm to calculate the normalized value of x_{ij} as defined in Eq. (2):

$$r_{ij} = \frac{x_{ij}}{\sqrt{\sum_{i=1}^{m} x_{ij}^2}}, \quad i = 1, \ldots, m; \; j = 1, \ldots, n. \quad (2)$$

Given the above terms, the formal TOPSIS procedure is defined as follows:

Step 1: Calculate normalized rating for each element in the decision matrix.

Step 2: Calculate weighted normalized ratings. The weighted normalized value v_{ij} is calculated by Eq. (3).

$$v_{ij} = w_j r_{ij}, \quad i = 1, \ldots, m; \; j = 1, \ldots, n. \quad (3)$$

Step 3: Identify positive ideal (A^*) and negative ideal (A^-) solutions. The A^* and A^- are defined in terms of the weighted normalized values, as shown in Eqs. (4) and (5), respectively:

$$A^* = \left\{ v_1^*, v_2^*, \ldots, v_j^*, \ldots, v_n^* \right\} = \{(\max_i v_{ij} | j \in J_1), (\min_i v_{ij} | j \in J_2) | i = 1, \ldots, m\}, \quad (4)$$

$$A^- = \left\{ v_1^-, v_2^-, \ldots, v_j^-, \ldots, v_n^- \right\} = \{(\min_i v_{ij} | j \in J_1), (\max_i vi_j | j \in J_2) | i = 1, \ldots, m\}, \quad (5)$$

where J_1 is a set of benefit attributes (larger-the-better type) and J_2 is a set of cost attributes (smaller-the-better type).

Step 4: Calculate separation measures. The separation (distance) between alternatives can be measured by the n-dimensional Euclidean distance. The separation of each alternative from the positive ideal solution, A^*, is given by Eq. (6):

$$S_i^* = \sqrt{\sum_{j=1}^{n} \left(v_{ij} - v_j^*\right)^2}, \quad i = 1, \ldots, m. \tag{6}$$

Similarly, the separation from the negative ideal solution, A^-, is given by Eq. (7):

$$S_i^- = \sqrt{\sum_{j=1}^{n} \left(v_{ij} - v_j^-\right)^2}, \quad i = 1, \ldots, m. \tag{7}$$

Step 5: Calculate similarities to ideal solution. This is defined in Eq. (8):

$$C_i^* = \frac{S_i^-}{S_i^* + S_i^-}, \quad i = 1, \ldots, m. \tag{8}$$

Note that $0 \leqslant C_i^* \leqslant 1$, where $C_i^* = 0$ when $A_i = A^-$, and $C_i^* = 1$ when $A_i = A^*$.

Step 6: Rank preference order. Choose an alternative with maximum C_i* or rank alternatives according to C_i* in descending order.

4.2. Empirical illustrations for TOPSIS method

The decision matrix from Table 2 is used for the TOPSIS analysis. Based on the first step of the TOPSIS procedure, each element is normalized by Eq. (2). The resulting normalized decision matrix for the TOPSIS analysis is shown as Table 3.

The second step requires the attribute weight information to calculate the weighted normalized ratings. The decision for the attribute weighting is often ambiguous; therefore we adopt the numeric scale method proposed by Ribeiro [21]. It uses a five grade scale from "extremely important (the grade of 5)" to "extremely unimportant (the grade of 1)". The calculation algorithm is shown as Eq. (9):

$$w_j = \frac{grade_j}{\sum_{j=1}^{n} grade_j}, \quad j = 1, 2, \ldots, n, \tag{9}$$

where $grade_j$ is the grade scale for attribute C_j. According to experts' opinion, the grade scales for the six attributes are {4, 4, 3, 2, 4, 3}. We collected a pretty unanimous conclusion during the weight data collection process, and thus, do not feel the compelling need to develop a more sophisticated approach. Then, the resulting numeric scale weights using Eq. (9) are shown as Eq. (10).

$$\begin{aligned} W &= \{4/20, 4/20, 3/20, 2/20, 4/20, 3/20\} \\ &= \{0.20, 0.20, 0.15, 0.10, 0.20, 0.15\}. \end{aligned} \tag{10}$$

The third step finds the weighted normalized decision matrix. The analysis then proceeds to Steps 4 and 5. The results are summarized in Table 4.

Finally, the sixth step ranks the alternative according to Table 4 results as follows:

$$A_{11} > A_{15} > A_{10} > A_4 > A_{14} > A_6 > A_{17} > A_{16} > A_2 > A_3 > A_{18} > A_5 > A_8 > A_{13} > A_9 > A_1 > A_{12} > A_7.$$

Table 3
Normalized decision matrix for TOPSIS analysis

No.	C_1	C_2	C_3	C_4	C_5	C_6
A_1	0.2137	0.2277	0.2311	0.0472	0.0978	0.2771
A_2	0.2383	0.2562	0.1047	0.2362	0.0978	0.2309
A_3	0.2372	0.2277	0.2191	0.2835	0.1952	0.1385
A_4	0.2180	0.2277	0.2311	0.2835	0.2931	0.1847
A_5	0.2430	0.2277	0.2152	0.2835	0.1467	0.1847
A_6	0.3035	0.1423	0.0578	0.1417	0.1952	0.2771
A_7	0.2620	0.2277	0.3908	0.1890	0.1467	0.0924
A_8	0.2133	0.2562	0.1745	0.1890	0.0489	0.2309
A_9	0.2136	0.2562	0.2191	0.1417	0.0978	0.2309
A_{10}	0.2714	0.2277	0.2191	0.2362	0.2931	0.2771
A_{11}	0.2105	0.2277	0.0558	0.3780	0.4398	0.3694
A_{12}	0.2347	0.2277	0.3713	0.1417	0.1467	0.2309
A_{13}	0.2589	0.2277	0.2272	0.2835	0.1467	0.1385
A_{14}	0.2331	0.2277	0.2233	0.1417	0.2931	0.2309
A_{15}	0.1955	0.2562	0.2311	0.3780	0.4398	0.3694
A_{16}	0.2487	0.2562	0.2152	0.1890	0.1952	0.2771
A_{17}	0.2066	0.2277	0.2875	0.1890	0.2931	0.1385
A_{18}	0.2135	0.2847	0.2836	0.2362	0.1952	0.1385
w_j	0.2000	0.2000	0.1500	0.1000	0.2000	0.1500

Table 4
TOPSIS analysis results

	v_{i1}	v_{i2}	v_{i3}	v_{i4}	v_{i5}	v_{i6}	S_i^*	S_i^-	C_i^*
A_1	0.0427	0.0455	0.0347	0.0047	0.0196	0.0416	0.0034	0.0010	0.2273
A_2	0.0477	0.0512	0.0157	0.0236	0.0196	0.0346	0.0027	0.0017	0.3864
A_3	0.0474	0.0455	0.0329	0.0283	0.0390	0.0208	0.0022	0.0013	0.3714
A_4	0.0436	0.0455	0.0347	0.0283	0.0586	0.0277	0.0013	0.0021	0.6176
A_5	0.0486	0.0455	0.0323	0.0283	0.0293	0.0277	0.0025	0.0011	0.3056
A_6	0.0607	0.0285	0.0087	0.0142	0.0390	0.0416	0.0022	0.0021	0.4884
A_7	0.0524	0.0455	0.0586	0.0189	0.0293	0.0139	0.0042	0.0005	0.1064
A_8	0.0427	0.0512	0.0262	0.0189	0.0098	0.0346	0.0036	0.0013	0.2653
A_9	0.0427	0.0512	0.0329	0.0142	0.0196	0.0346	0.0032	0.0011	0.2558
A_{10}	0.0543	0.0455	0.0329	0.0236	0.0586	0.0416	0.0011	0.0023	0.6765
A_{11}	0.0421	0.0455	0.0084	0.0378	0.0880	0.0554	0.0001	0.0060	0.9836
A_{12}	0.0469	0.0455	0.0557	0.0142	0.0293	0.0346	0.0034	0.0007	0.1707
A_{13}	0.0518	0.0455	0.0341	0.0283	0.0293	0.0208	0.0028	0.0010	0.2632
A_{14}	0.0466	0.0455	0.0335	0.0142	0.0586	0.0346	0.0013	0.0020	0.6061
A_{15}	0.0391	0.0512	0.0347	0.0378	0.0880	0.0554	0.0004	0.0052	0.9286
A_{16}	0.0497	0.0512	0.0323	0.0189	0.0390	0.0416	0.0018	0.0016	0.4706
A_{17}	0.0413	0.0455	0.0431	0.0189	0.0586	0.0208	0.0019	0.0018	0.4865
A_{18}	0.0427	0.0569	0.0425	0.0236	0.0390	0.0208	0.0025	0.0013	0.3421
v_j^*	0.0391	0.0569	0.0084	0.0378	0.0880	0.0554			
v_j	0.0607	0.0285	0.0586	0.0047	0.0098	0.0139			

$W = (0.2, 0.2, 0.15, 0.1, 0.2, 0.15)$

Eventually, only the 'best' will be adopted as the final design. Our attention should focus on the top few choices if there is a need for further investigation or discussion.

5. Fuzzy TOPSIS

5.1. Fuzzy TOPSIS model

It is often difficult for a decision-maker to assign a precise performance rating to an alternative for the attributes under consideration. The merit of using a fuzzy approach is to assign the relative importance of attributes using fuzzy numbers instead of precise numbers. This section extends the TOPSIS to the fuzzy environment. This method is particularly suitable for solving the group decision-making problem under fuzzy environment. We briefly review the rationale of fuzzy theory before the development of fuzzy TOPSIS; as follows:

Definition 5.1. A fuzzy set $\tilde{a}$ in a universe of discourse X is characterized by a membership function $\mu_{\tilde{a}}(x)$ which associates with each element x in X, a real number in the interval [0, 1]. The function value $\mu_{\tilde{a}}(x)$ is termed the grade of membership of x in $\tilde{a}$ [31].

The present study uses triangular fuzzy numbers. A triangular fuzzy number $\tilde{a}$ can be defined by a triplet (a_1, a_2, a_3). Its conceptual schema and mathematical form are shown by Eq. (11) [32]:

$$\mu_{\tilde{a}}(x) = \begin{cases} 0, & x \leqslant a_1, \\ \frac{x-a_1}{a_2-a_1}, & a_1 < x \leqslant a_2, \\ \frac{a_3-x}{a_3-a_2}, & a_2 < x \leqslant a_3, \\ 0, & x > a_3, \end{cases} \quad (11)$$

Definition 5.2. Let $\tilde{a} = (a_1, a_2, a_3)$ and $\tilde{b} = (b_1, b_2, b_3)$ be two triangular fuzzy numbers, then the vertex method is defined to calculate the distance between them, as Eq. (12):

$$d(\tilde{a}, \tilde{b}) = \sqrt{\frac{1}{3}\left[(a_1 - b_1)^2 + (a_2 - b_2)^2 + (a_3 - b_3)^2\right]}. \quad (12)$$

Property 5.1. *Assuming that both $\tilde{a} = (a_1, a_2, a_3)$ and $\tilde{b} = (b_1, b_2, b_3)$ are real numbers, then the distance measurement $d(\tilde{a}, \tilde{b})$ is identical to the Euclidean distance* [33].

Property 5.2. *Let $\tilde{a}$, $\tilde{b}$, and $\tilde{c}$ be three triangular fuzzy numbers. The fuzzy number $\tilde{b}$ is closer to fuzzy number $\tilde{a}$ than the other fuzzy number $\tilde{c}$ if, and only if, $d(\tilde{a}, \tilde{b}) < d(\tilde{a}, \tilde{c})$* [33].

The basic operations on fuzzy triangular numbers are as follows:

$$\tilde{a} \times \tilde{b} = (a_1 \times b_1, a_2 \times b_2, a_3 \times b_3) \text{ for multiplication,} \quad (13)$$

$$\tilde{a} + \tilde{b} = (a_1 + b_1, a_2 + b_2, a_3 + b_3) \text{ for addition.} \quad (14)$$

The fuzzy MADM can be concisely expressed in matrix format as Eqs. (15) and (16).

$$\tilde{D} = \begin{array}{c} \\ A_1 \\ A_2 \\ A_3 \\ \vdots \\ A_m \end{array} \begin{array}{c} \begin{array}{ccccc} C_1 & C_2 & C_3 & \dots & C_n \end{array} \\ \begin{bmatrix} \tilde{x}_{11} & \tilde{x}_{12} & \tilde{x}_{13} & \dots & \tilde{x}_{1n} \\ \tilde{x}_{21} & \tilde{x}_{22} & \tilde{x}_{23} & \dots & \tilde{x}_{2n} \\ \tilde{x}_{31} & \tilde{x}_{32} & \tilde{x}_{33} & \dots & \tilde{x}_{3n} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \tilde{x}_{m1} & \tilde{x}_{m2} & \tilde{x}_{m3} & \dots & \tilde{x}_{mn} \end{bmatrix} \end{array}, \quad (15)$$

$$\tilde{W} = [\tilde{w}_1, \tilde{w}_2, \dots, \tilde{w}_n], \quad (16)$$

where $\tilde{x}_{ij}$, $i = 1, 2,\dots, m, j = 1, 2,\dots, n$ and $\tilde{w}_j$, $j = 1, 2,\dots, n$ are linguistic triangular fuzzy numbers, $\tilde{x}_{ij} = (a_{ij}, b_{ij}, c_{ij})$ and $\tilde{w}_j = (w_{j1}, w_{j2}, w_{j3})$. Note that $\tilde{x}_{ij}$ is the performance rating of the ith alternative, A_i, with respect to the jth attribute, C_j and $\tilde{w}_j$ represents the weight of the jth attribute, C_j.

The normalized fuzzy decision matrix denoted by $\tilde{R}$ is shown as Eq. (17):

$$\tilde{R} = [\tilde{r}_{ij}]_{m \times n}. \quad (17)$$

The weighted fuzzy normalized decision matrix is shown as Eq. (18):

$$\tilde{V} = \begin{bmatrix} \tilde{v}_{11} & \tilde{v}_{12} & \cdots & \tilde{v}_{1j} & \cdots & \tilde{v}_{1n} \\ \tilde{v}_{21} & \tilde{v}_{22} & \cdots & \tilde{v}_{2j} & \cdots & \tilde{v}_{2n} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ \tilde{v}_{i1} & \tilde{v}_{i2} & \cdots & \tilde{v}_{ij} & \cdots & \tilde{v}_{in} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ \tilde{v}_{m1} & \tilde{v}_{m2} & \cdots & \tilde{v}_{mj} & \cdots & \tilde{v}_{mn} \end{bmatrix} = \begin{bmatrix} \tilde{w}_1\tilde{r}_{11} & \tilde{w}_2\tilde{r}_{12} & \cdots & \tilde{w}_j\tilde{r}_{1j} & \cdots & \tilde{w}_n\tilde{r}_{1n} \\ \tilde{w}_1\tilde{r}_{21} & \tilde{w}_2\tilde{r}_{22} & \cdots & \tilde{w}_j\tilde{r}_{2j} & \cdots & \tilde{w}_n\tilde{r}_{2n} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ \tilde{w}_1\tilde{r}_{i1} & \tilde{w}_2\tilde{r}_{i2} & \cdots & \tilde{w}_j\tilde{r}_{ij} & \cdots & \tilde{w}_n\tilde{r}_{in} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ \tilde{w}_1\tilde{r}_{m1} & \tilde{w}_2\tilde{r}_{m2} & \cdots & \tilde{w}_j\tilde{r}_{mj} & \cdots & \tilde{w}_n\tilde{r}_{mn} \end{bmatrix}. \quad (18)$$

Given the above fuzzy theory, the proposed fuzzy TOPSIS procedure is then defined as follows:

Step 1: Choose the linguistic ratings ($\tilde{x}_{ij}$, $i = 1, 2,\dots, m, j = 1, 2,\dots, n$) for alternatives with respect to criteria and the appropriate linguistic variables ($\tilde{w}_j$, $j = 1, 2,\dots, n$) for the weight of the criteria.

The fuzzy linguistic rating ($\tilde{x}_{ij}$) preserves the property that the ranges of normalized triangular fuzzy numbers belong to [0,1]; thus, there is no need for a normalization procedure. For this instance, the $\tilde{D}$ defined by Eq. (15) is equivalent to the $\tilde{R}$ defined by Eq. (17).

Step 2: Construct the weighted normalized fuzzy decision matrix. The weighted normalized value $\tilde{V}$ is calculated by Eq. (18).

Step 3: Identify positive ideal (A^*) and negative ideal (A^-) solutions. The fuzzy positive-ideal solution (FPIS, A^*) and the fuzzy negative-ideal solution (FNIS, A^-) are shown as Eqs. (19) and (20):

$$A^* = \left(\tilde{v}_1^*, \tilde{v}_2^*, \dots, \tilde{v}_n^*\right) = \{(\max_i v_{ij} | i = 1, 2, \dots, m), j = 1, 2, \dots, n\}, \quad (19)$$

$$A^- = \left(\tilde{v}_1^-, \tilde{v}_2^-, \dots, \tilde{v}_n^-\right) = \{(\min_i v_{ij} | i = 1, 2, \dots, m), j = 1, 2, \dots, n\}. \quad (20)$$

Step 4: Calculate separation measures. The distance of each alternative from A^* and A^- can be currently calculated using Eqs. (21) and (22).

$$d_i^* = \sum_{j=1}^{n} d\left(\tilde{v}_{ij}, \tilde{v}_j^*\right), i = 1, 2, \dots, m \quad (21)$$

$$d_i^- = \sum_{j=1}^{n} d\left(\tilde{v}_{ij}, \tilde{v}_j^-\right), i = 1, 2, \dots, m \quad (22)$$

Step 5: Calculate similarities to ideal solution. This step solves the similarities to an ideal solution by Eq. (23):

$$CC_i = \frac{d_i^-}{d_i^* + d_i^-}. \quad (23)$$

Step 6: Rank preference order. Choose an alternative with maximum CC_i^* or rank alternatives according to CC_i^* in descending order.

The proposed fuzzy TOPSIS is then applied to the case study as shown in Sections 5.2 and 5.3.

5.2. Fuzzy membership function

The decision makers use the linguistic variables to evaluate the importance of attributes and the ratings of alternatives with respect to various attributes. The present study has only precise values for the performance ratings and for the attribute weights. In order to illustrate the idea of fuzzy MACD, we deliberately transform the existing precise values to five-levels, fuzzy linguistic variables—very low (VL), low (L), medium (M), high (H) and very high (VH). The purpose of the transformation process has two folds as: (i) to illustrate the proposed fuzzy TOPSIS method and (ii) to benchmark the empirical results with other precise value methods in the later analysis.

Among the commonly used fuzzy numbers, triangular and trapezoidal fuzzy numbers are likely to be the most adoptive ones due to their simplicity in modeling and easy of interpretation. Both triangular and trapezoidal fuzzy numbers are applicable to the present study. We feel that a triangular fuzzy number can adequately represent the five-level fuzzy linguistic variables and thus, is used for the analysis hereafter.

As a rule of thumb, each rank is assigned an evenly spread membership function that has an interval of 0.30 or 0.25. Based on these assumptions, a transformation table can be found as shown in Table 5. For example, the fuzzy variable—Very Low has its associated triangular fuzzy number with minimum of 0.00, mode of 0.10 and maximum of 0.25. The same definition is then applied to the other fuzzy variables—Low, Medium, High and Very High. Fig. 1 illustrates the fuzzy membership functions.

5.3. Empirical illustrations

Table 2 numeric performance ratings are adopted again for the fuzzy TOPSIS analysis. In order to transform the performance ratings to fuzzy linguistic variables as discussed in the previous section, the performance ratings in Table 2 are normalized into the range of [0,1] by Eqs. (24) and (25) [34]:

(i) The larger the better type:

$$r_{ij} = \left[x_{ij} - \min\{x_{ij}\}\right] / \left[\max\{x_{ij}\} - \min\{x_{ij}\}\right]. \qquad (24)$$

(ii) The smaller the better type:

$$r_{ij} = \left[\max\{x_{ij}\} - x_{ij}\right] / \left[\max\{x_{ij}\} - \min\{x_{ij}\}\right]. \qquad (25)$$

For the present study, C_1 and C_3 are the smaller-the-better type, the others belong to the larger-the-better type. Then, Table 2 can be transformed into Table 6.

Table 5
Transformation for fuzzy membership functions

Rank	Attribute grade	Membership functions
Very low (VL)	1	(0.00, 0.10, 0.25)
Low (L)	2	(0.15, 0.30, 0.45)
Medium (M)	3	(0.35, 0.50, 0.65)
High (H)	4	(0.55, 0.70, 0.85)
Very high (VH)	5	(0.75, 0.90, 1.00)

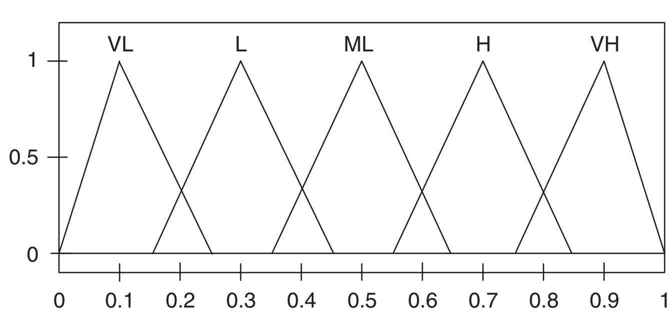

Fig. 1. Fuzzy triangular membership functions.

Table 6
Normalized decision matrix for fuzzy TOPSIS analysis

No.	C_1	C_2	C_3	C_4	C_5	C_6
A_1	0.76	0.60	0.12	0.10	0.18	0.71
A_2	0.49	0.80	0.46	0.60	0.18	0.57
A_3	0.51	0.60	0.13	0.70	0.36	0.25
A_4	0.71	0.60	0.12	0.70	0.64	0.43
A_5	0.45	0.60	0.14	0.70	0.27	0.43
A_6	0.00	0.00	0.96	0.40	0.36	0.71
A_7	0.29	0.60	0.00	0.50	0.27	0.00
A_8	0.77	0.80	0.21	0.50	0.00	0.57
A_9	0.76	0.80	0.13	0.40	0.18	0.57
A_{10}	0.21	0.60	0.13	0.60	0.64	0.71
A_{11}	0.80	0.60	1.00	1.00	1.00	1.00
A_{12}	0.53	0.60	0.01	0.40	0.27	0.57
A_{13}	0.31	0.60	0.12	0.70	0.27	0.14
A_{14}	0.55	0.60	0.13	0.40	0.64	0.57
A_{15}	1.00	0.80	0.12	1.00	1.00	1.00
A_{16}	0.40	0.80	0.14	0.50	0.36	0.71
A_{17}	0.76	0.60	0.12	0.10	0.18	0.71
A_{18}	0.49	0.80	0.46	0.60	0.18	0.57
w_j	0.2000	0.2000	0.1500	0.1000	0.2000	0.1500

Table 7
Decision matrix using fuzzy linguistic variables

No.	C_1	C_2	C_3	C_4	C_5	C_6
A_1	H	M	VL	VL	VL	H
A_2	M	H	M	M	VL	M
A_3	M	M	VL	H	L	L
A_4	H	M	VL	H	M	M
A_5	M	M	VL	H	L	M
A_6	VL	VL	VH	L	L	H
A_7	L	M	VL	M	L	VL
A_8	H	H	VL	M	VL	M
A_9	H	H	VL	L	VL	M
A_{10}	VL	M	VL	M	M	H
A_{11}	VH	H	VH	VH	VH	VH
A_{12}	M	M	VL	L	L	M
A_{13}	L	M	VL	H	L	VL
A_{14}	M	M	VL	L	M	M
A_{15}	VH	H	VL	VH	VH	VH
A_{16}	L	H	VL	M	L	H
A_{17}	H	M	VL	M	M	VL
A_{18}	VH	VH	VL	H	M	VL
Weight	H	H	M	L	H	M

The next step uses the fuzzy membership function discussed in Section 5.2 to transform Table 6 into Table 7 as explained by the following example. If the numeric rating is 0.64, then its fuzzy linguistic variable is "H". This transformation is also applied to the attribute weight $W = \{0.20, 0.20, 0.15, 0.10, 0.20, 0.15\}$ for C_1, C_2, C_3, C_4, C_5, C_6, respectively. Then, the resulting fuzzy linguistic variables are show as Table 7.

The fuzzy linguistic variable is then transformed into a fuzzy triangular membership function as shown in Table 8. This is *the first step* of the fuzzy TOPSIS analysis. The fuzzy attribute weight is also collected in Table 8.

The second step in the analysis is to find the weighted fuzzy decision matrix. Using Eq. (13), the fuzzy multiplication equation, the resulting fuzzy weighted decision matrix is shown as Table 9.

According to Table 9, we know that the elements $\tilde{v}_{ij}$, $\forall i,j$ are normalized positive triangular fuzzy numbers and their ranges belong to the closed interval [0,1]. Thus, we can define the fuzzy positive-ideal solution (FPIS, A^*) and the fuzzy negative-ideal solution (FNIS, A^-) as: $\tilde{v}_j^* = (1,1,1)$ and $\tilde{v}_j^- = (0,0,0), j = 1, 2, \ldots, n$. This is *the third step* of the fuzzy TOPSIS analysis.

For *the fourth step*, the distance of each alternative from A^* and A^- can be currently calculated using Eqs. (21) and (22). *The fifth step* solves the similarities to an ideal solution by Eq. (23). The resulting fuzzy TOPSIS analyses are summarized in Table 10.

Table 8
Fuzzy decision matrix and fuzzy attribute weights

No.	C_1	C_2	C_3	C_4	C_5	C_6
A_1	(0.55, 0.70, 0.85)	(0.35, 0.50, 0.65)	(0.00, 0.10, 0.25)	(0.00, 0.10, 0.25)	(0.00, 0.10, 0.25)	(0.55, 0.70, 0.85)
A_2	(0.35, 0.50, 0.65)	(0.55, 0.70, 0.85)	(0.35, 0.50, 0.65)	(0.35, 0.50, 0.65)	(0.00, 0.10, 0.25)	(0.35, 0.50, 0.65)
A_3	(0.35, 0.50, 0.65)	(0.35, 0.50, 0.65)	(0.00, 0.10, 0.25)	(0.55, 0.70, 0.85)	(0.15, 0.30, 0.45)	(0.15, 0.30, 0.45)
A_4	(0.55, 0.70, 0.85)	(0.35, 0.50, 0.65)	(0.00, 0.10, 0.25)	(0.55, 0.70, 0.85)	(0.55, 0.70, 0.85)	(0.35, 0.50, 0.65)
A_5	(0.35, 0.50, 0.65)	(0.35, 0.50, 0.65)	(0.00, 0.10, 0.25)	(0.55, 0.70, 0.85)	(0.15, 0.30, 0.45)	(0.35, 0.50, 0.65)
A_6	(0.00, 0.10, 0.25)	(0.00, 0.10, 0.25)	(0.75, 0.90, 1.00)	(0.15, 0.30, 0.45)	(0.15, 0.30, 0.45)	(0.55, 0.70, 0.85)
A_7	(0.15, 0.30, 0.45)	(0.35, 0.50, 0.65)	(0.00, 0.10, 0.25)	(0.35, 0.50, 0.65)	(0.15, 0.30, 0.45)	(0.00, 0.10, 0.25)
A_8	(0.55, 0.70, 0.85)	(0.55, 0.70, 0.85)	(0.15, 0.30, 0.45)	(0.35, 0.50, 0.65)	(0.00, 0.10, 0.25)	(0.35, 0.50, 0.65)
A_9	(0.55, 0.70, 0.85)	(0.55, 0.70, 0.85)	(0.00, 0.10, 0.25)	(0.15, 0.30, 0.45)	(0.00, 0.10, 0.25)	(0.35, 0.50, 0.65)
A_{10}	(0.15, 0.30, 0.45)	(0.35, 0.50, 0.65)	(0.00, 0.10, 0.25)	(0.35, 0.50, 0.65)	(0.55, 0.70, 0.85)	(0.55, 0.70, 0.85)
A_{11}	(0.75, 0.90, 1.00)	(0.55, 0.70, 0.85)	(0.75, 0.90, 1.00)	(0.75, 0.90, 1.00)	(0.75, 0.90, 1.00)	(0.75, 0.90, 1.00)
A_{12}	(0.35, 0.50, 0.65)	(0.35, 0.50, 0.65)	(0.00, 0.10, 0.25)	(0.15, 0.30, 0.45)	(0.15, 0.30, 0.45)	(0.35, 0.50, 0.65)
A_{13}	(0.15, 0.30, 0.45)	(0.35, 0.50, 0.65)	(0.00, 0.10, 0.25)	(0.55, 0.70, 0.85)	(0.15, 0.30, 0.45)	(0.00, 0.10, 0.25)
A_{14}	(0.35, 0.50, 0.65)	(0.35, 0.50, 0.65)	(0.00, 0.10, 0.25)	(0.15, 0.30, 0.45)	(0.55, 0.70, 0.85)	(0.35, 0.50, 0.65)
A_{15}	(0.75, 0.90, 1.00)	(0.55, 0.70, 0.85)	(0.00, 0.10, 0.25)	(0.75, 0.90, 1.00)	(0.75, 0.90, 1.00)	(0.75, 0.90, 1.00)
A_{16}	(0.15, 0.30, 0.45)	(0.55, 0.70, 0.85)	(0.00, 0.10, 0.25)	(0.35, 0.50, 0.65)	(0.15, 0.30, 0.45)	(0.55, 0.70, 0.85)
A_{17}	(0.75, 0.90, 1.00)	(0.35, 0.50, 0.65)	(0.00, 0.10, 0.25)	(0.35, 0.50, 0.65)	(0.55, 0.70, 0.85)	(0.00, 0.10, 0.25)
A_{18}	(0.55, 0.70, 0.85)	(0.75, 0.90, 1.00)	(0.00, 0.10, 0.25)	(0.55, 0.70, 0.85)	(0.15, 0.30, 0.45)	(0.00, 0.10, 0.25)
Weight	(0.55, 0.70, 0.85)	(0.55, 0.70, 0.85)	(0.35, 0.50, 0.65)	(0.15, 0.30, 0.45)	(0.55, 0.70, 0.85)	(0.35, 0.50, 0.65)

Table 9
Fuzzy-weighted decision matrix

No.	C_1	C_2	C_3	C_4	C_5	C_6
A_1	(0.30, 0.49, 0.72)	(0.19, 0.35, 0.55)	(0.00, 0.05, 0.16)	(0.00, 0.03, 0.11)	(0.00, 0.07, 0.21)	(0.19, 0.35, 0.55)
A_2	(0.19, 0.35, 0.55)	(0.30, 0.49, 0.72)	(0.12, 0.25, 0.42)	(0.05, 0.15, 0.29)	(0.00, 0.07, 0.21)	(0.12, 0.25, 0.42)
A_3	(0.19, 0.35, 0.55)	(0.19, 0.35, 0.55)	(0.00, 0.05, 0.16)	(0.08, 0.21, 0.38)	(0.08, 0.21, 0.38)	(0.05, 0.15, 0.29)
A_4	(0.30, 0.49, 0.72)	(0.19, 0.35, 0.55)	(0.00, 0.05, 0.16)	(0.08, 0.21, 0.38)	(0.19, 0.35, 0.55)	(0.12, 0.25, 0.42)
A_5	(0.19, 0.35, 0.55)	(0.19, 0.35, 0.55)	(0.00, 0.05, 0.16)	(0.08, 0.21, 0.38)	(0.08, 0.21, 0.38)	(0.12, 0.25, 0.42)
A_6	(0.00, 0.07, 0.21)	(0.00, 0.07, 0.21)	(0.26, 0.45, 0.65)	(0.02, 0.09, 0.20)	(0.08, 0.21, 0.38)	(0.19, 0.35, 0.55)
A_7	(0.08, 0.21, 0.38)	(0.19, 0.35, 0.55)	(0.00, 0.05, 0.16)	(0.05, 0.15, 0.29)	(0.08, 0.21, 0.38)	(0.05, 0.15, 0.29)
A_8	(0.30, 0.49, 0.72)	(0.30, 0.49, 0.72)	(0.00, 0.05, 0.16)	(0.05, 0.15, 0.29)	(0.00, 0.07, 0.21)	(0.12, 0.25, 0.42)
A_9	(0.30, 0.49, 0.72)	(0.30, 0.49, 0.72)	(0.00, 0.05, 0.16)	(0.02, 0.09, 0.20)	(0.00, 0.07, 0.21)	(0.12, 0.25, 0.42)
A_{10}	(0.00, 0.07, 0.21)	(0.19, 0.35, 0.55)	(0.00, 0.05, 0.16)	(0.05, 0.15, 0.29)	(0.19, 0.35, 0.55)	(0.19, 0.35, 0.55)
A_{11}	(0.41, 0.63, 0.85)	(0.30, 0.49, 0.72)	(0.26, 0.45, 0.65)	(0.11, 0.27, 0.45)	(0.41, 0.63, 0.85)	(0.26, 0.45, 0.65)
A_{12}	(0.19, 0.35, 0.55)	(0.19, 0.35, 0.55)	(0.00, 0.05, 0.16)	(0.02, 0.09, 0.20)	(0.08, 0.21, 0.38)	(0.12, 0.25, 0.42)
A_{13}	(0.08, 0.21, 0.38)	(0.19, 0.35, 0.55)	(0.00, 0.05, 0.16)	(0.08, 0.21, 0.38)	(0.08, 0.21, 0.38)	(0.00, 0.05, 0.16)
A_{14}	(0.19, 0.35, 0.55)	(0.19, 0.35, 0.55)	(0.00, 0.05, 0.16)	(0.02, 0.09, 0.20)	(0.19, 0.35, 0.55)	(0.12 ,0.25, 0.42)
A_{15}	(0.41, 0.63, 0.85)	(0.30, 0.49, 0.72)	(0.00, 0.05, 0.16)	(0.11, 0.27, 0.45)	(0.41, 0.63, 0.85)	(0.26, 0.45, 0.65)
A_{16}	(0.08, 0.21, 0.38)	(0.30, 0.49, 0.72)	(0.00, 0.05, 0.16)	(0.05, 0.15, 0.29)	(0.08, 0.21, 0.38)	(0.19, 0.35, 0.55)
A_{17}	(0.30, 0.49, 0.72)	(0.19, 0.35, 0.55)	(0.00, 0.05, 0.16)	(0.05, 0.15, 0.29)	(0.19, 0.35, 0.55)	(0.00, 0.05, 0.16)
A_{18}	(0.41, 0.63, 0.85)	(0.41, 0.63, 0.85)	(0.00, 0.05, 0.16)	(0.08, 0.21, 0.38)	(0.19, 0.35, 0.55)	(0.00, 0.05, 0.16)

Table 10
Fuzzy TOPSIS analysis

No.	$\hat{v}_{i1}$	$\hat{v}_{i2}$	$\hat{v}_{i3}$	$\hat{v}_{i4}$	$\hat{v}_{i5}$	$\hat{v}_{i6}$	d_i^+	d_i^-	CC_i
A_1	(0.30, 0.49, 0.72)	(0.19, 0.35, 0.55)	(0.00, 0.05, 0.16)	(0.00, 0.03, 0.11)	(0.00, 0.07, 0.21)	(0.19, 0.35, 0.55)	4.635148	1.606324	0.257363
A_2	(0.19, 0.35, 0.55)	(0.30, 0.49, 0.72)	(0.12, 0.25, 0.42)	(0.05, 0.15, 0.29)	(0.00, 0.07, 0.21)	(0.12, 0.25, 0.42)	4.425966	1.823517	0.291787
A_3	(0.19, 0.35, 0.55)	(0.19, 0.35, 0.55)	(0.00, 0.05, 0.16)	(0.08, 0.21, 0.38)	(0.08, 0.21,0.38)	(0.05, 0.15, 0.29)	4.654412	1.581345	0.253592
A_4	(0.30, 0.49, 0.72)	(0.19, 0.35, 0.55)	(0.00, 0.05, 0.16)	(0.08, 0.21, 0.38)	(0.19, 0.35, 0.55)	(0.12, 0.25, 0.42)	4.298028	1.958154	0.312995
A_5	(0.19, 0.35, 0.55)	(0.19, 0.35, 0.55)	(0.00, 0.05, 0.16)	(0.08, 0.21, 0.38)	(0.08, 0.21, 0.38)	(0.12, 0.25, 0.42)	4.558833	1.681219	0.269424
A_6	(0.00, 0.07, 0.21)	(0.00, 0.07, 0.21)	(0.26, 0.45, 0.65)	(0.02, 0.09, 0.20)	(0.08, 0.21, 0.38)	(0.19, 0.35, 0.55)	4.730627	1.510169	0.241983
A_7	(0.08, 0.21, 0.38)	(0.19, 0.35, 0.55)	(0.00, 0.05, 0.16)	(0.05, 0.15, 0.29)	(0.08, 0.21, 0.38)	(0.05, 0.15, 0.29)	4.843391	1.510169	0.221744
A_8	(0.30, 0.49, 0.72)	(0.30, 0.49, 0.72)	(0.00, 0.05, 0.16)	(0.05, 0.15, 0.29)	(0.00, 0.07, 0.21)	(0.12, 0.25, 0.42)	4.483565	1.769500	0.282981
A_9	(0.30, 0.49, 0.72)	(0.30, 0.49, 0.72)	(0.00, 0.05, 0.16)	(0.02, 0.09, 0.20)	(0.00, 0.07, 0.21)	(0.12, 0.25, 0.42)	4.54085	1.705947	0.273092
A_{10}	(0.00, 0.07, 0.21)	(0.19, 0.35, 0.55)	(0.00, 0.05, 0.16)	(0.05, 0.15, 0.29)	(0.19, 0.35, 0.55)	(0.19, 0.35, 0.55)	4.646129	1.591422	0.255136
A_{11}	(0.41, 0.63, 0.85)	(0.30, 0.49, 0.72)	(0.26, 0.45, 0.65)	(0.11, 0.27, 0.45)	(0.41, 0.63, 0.85)	(0.26, 0.45, 0.65)	3.22343	3.112578	0.491252
A_{12}	(0.19, 0.35, 0.55)	(0.19, 0.35, 0.55)	(0.00, 0.05, 0.16)	(0.02, 0.09, 0.20)	(0.08, 0.21, 0.38)	(0.12, 0.25, 0.42)	4.672235	1.553481	0.249527
A_{13}	(0.08, 0.21, 0.38)	(0.19, 0.35, 0.55)	(0.00, 0.05, 0.16)	(0.08, 0.21, 0.38)	(0.08, 0.21, 0.38)	(0.00, 0.05, 0.16)	4.877236	1.350266	0.216823
A_{14}	(0.19, 0.35, 0.55)	(0.19, 0.35, 0.55)	(0.00, 0.05, 0.16)	(0.02, 0.09, 0.20)	(0.19, 0.35, 0.55)	(0.12, 0.25, 0.42)	4.539392	1.690642	0.27137
A_{15}	(0.41, 0.63, 0.85)	(0.30, 0.49, 0.72)	(0.00, 0.05, 0.16)	(0.11, 0.27, 0.45)	(0.41, 0.63, 0.85)	(0.26, 0.45, 0.65)	3.586443	2.728874	0.432104
A_{16}	(0.08, 0.21, 0.38)	(0.30, 0.49, 0.72)	(0.00, 0.05, 0.16)	(0.05, 0.15, 0.29)	(0.08, 0.21, 0.38)	(0.19, 0.35, 0.55)	4.52647	1.721120	0.275485
A_{17}	(0.30, 0.49, 0.72)	(0.19, 0.35, 0.55)	(0.00, 0.05, 0.16)	(0.05, 0.15, 0.29)	(0.19, 0.35, 0.55)	(0.00, 0.05, 0.16)	4.539705	1.700176	0.272469
A_{18}	(0.41, 0.63, 0.85)	(0.41, 0.63, 0.85)	(0.00, 0.05, 0.16)	(0.08, 0.21, 0.38)	(0.19, 0.35, 0.55)	(0.00, 0.05, 0.16)	4.127193	2.150711	0.342584
A^*	$\tilde{v}_1^* = (1,1,1)$	$\tilde{v}_2^* = (1,1,1)$	$\tilde{v}_3^* = (1,1,1)$	$\tilde{v}_4^* = (1,1,1)$	$\tilde{v}_5^* = (1,1,1)$	$\tilde{v}_6^* = (1,1,1)$			
A^-	$\tilde{v}_1^- = (0,0,0)$	$\tilde{v}_2^- = (0,0,0)$	$\tilde{v}_3^- = (0,0,0)$	$\tilde{v}_4^- = (0,0,0)$	$\tilde{v}_5^- = (0,0,0)$	$\tilde{v}_6^- = (0,0,0)$			

$W = \{(0.55, 0.70, 0.85), (0.55, 0.70, 0.85), (0.35, 0.50, 0.65), (0.15, 0.30, 0.45), (0.55, 0.70, 0.85), (0.35, 0.50, 0.65)\}$

In order to illustrate Steps 4 and 5 calculation, CC_1 calculation is used as an example as follows:

$$\begin{aligned} d_1^* = {} & \sqrt{\frac{1}{3}[(1-0.3)^2+(1-0.49)^2+(1-0.72)^2]} \\ & + \sqrt{\frac{1}{3}[(1-0.19)^2+(1-0.35)^2+(1-0.55)^2]} \\ & + \sqrt{\frac{1}{3}[(1-0)^2+(1-0.05)^2+(1-0.16)^2]} \\ & + \sqrt{\frac{1}{3}[(1-0)^2+(1-0.03)^2+(1-0.11)^2]} \\ & + \sqrt{\frac{1}{3}[(1-0)^2+(1-0.07)^2+(1-0.21)^2]} \\ & + \sqrt{\frac{1}{3}[(1-0.19)^2+(1-0.35)^2+(1-0.55)^2]} \\ = {} & 4.635148, \end{aligned}$$

$$\begin{aligned} d_1^- = {} & \sqrt{\frac{1}{3}[(0-0.3)^2+(0-0.49)^2+(0-0.72)^2]} \\ & + \sqrt{\frac{1}{3}[(0-0.19)^2+(0-0.35)^2+(0-0.55)^2]} \\ & + \sqrt{\frac{1}{3}[(0-0)^2+(0-0.05)^2+(0-0.16)^2]} \\ & + \sqrt{\frac{1}{3}[(0-0)^2+(0-0.03)^2+(0-0.11)^2]} \\ & + \sqrt{\frac{1}{3}[(0-0)^2+(0-0.07)^2+(0-0.21)^2]} \\ & + \sqrt{\frac{1}{3}[(0-0.19)^2+(0-0.35)^2+(0-0.55)^2]} \\ = {} & 1.606324, \end{aligned}$$

$$\begin{aligned} CC_1 &= \frac{d_1^*}{d_1^* + d_1^-} \\ &= \frac{1.606324}{4.635148 + 1.606324} = 0.257363. \end{aligned}$$

Based on Table 10, *the last step* found the preference for the 18 design alternatives as follows:

$$\begin{aligned} & A_{11} > A_{15} > A_{18} > A_4 > A_{17} > A_8 > A_{10} > A_{14} > A_2 > A_{16} \\ & > A_9 > A_5 > A_1 > A_3 > A_{12} > A_6 > A_7 > A_{13}. \end{aligned}$$

In this section, we deliberately transform the existing precise values to fuzzy linguistic variables in order to illustrate the concept of the proposed fuzzy-based method. It is the aim of this section to illustrate the feasibility of the fuzzy-based method for the instance of fuzzy inputs, which is justified by the empirical results.

6. Discussion

The resulting layouts vary among the different design methods to some extent. The top five design alternatives, according to the two proposed design methods, as well as the results by DEA [10], are summarized in Table 11.

All methods lead to the choice of A_{11} a priori as the final layout design. A_{15} is apparently the second choice. Other than these two alternatives, the preferences vary between methods. The fuzzy TOPSIS concludes with the same top three alternatives as those DEA. The TOPSIS method concludes the same top two alternatives as the ones from DEA and fuzzy TOPSIS. Due to the MADM nature of the proposed problem, an optimal solution may not exist; however, the systematic evaluation of the MADM problem can reduce the risk of a poor design.

Table 11
Top five design alternatives from different methods

Preference order	1	2	3	4	5
TOPSIS	A_{11}	A_{15}	A_{10}	A_4	A_{14}
Fuzzy TOPSIS	A_{11}	A_{15}	A_{18}	A_4	A_{17}
DEA	A_{11}^*	A_{15}^*	A_{18}^*	A_2	A_{16}

*Tie.

When precise performance ratings are available, the TOPSIS method is considered to be a viable approach in solving a layout design problem. The DEA method from existing literature is also a viable approach. However, it has the constraints in the number of decision making units and in the limitation to the discrepancy between performance frontiers. For the instance of imprecise or vague performance ratings, the fuzzy TOPSIS is a preferred choice in solving the proposed design problem.

Although it is the aim of the proposed methodology to solve the optimal layout design problem, this goal is not achievable due to the nature of a layout design problem. In addition to the MADM nature, there are other constraints form the shop floor limitations and from the management philosophy. For instance, the qualitative performance measures are mainly collected from the industry experts. The group decision making can be complicated, and perhaps, may not have a unanimous result. Many practical limitations are not quantifiable. For another example, the management may have their preference against the analysis results. Nevertheless, the proposed methodology provides a systematic approach to narrow down the number of alternatives, and to facilitate the decision making process. In fact, the management of the case study company has adopted the analysis results as the basis to their future layout planning problem. The practical contribution of this research can, thus, be justified.

7. Conclusions

The layout design problem is a strategic issue and has significant impacts to the efficiency of a manufacturing system. Much of the existing layout design literature that uses a surrogate function for flow distance or for simplified objectives may be entrapped into local optimum; and therefore lead to a poor layout design due to the MADM nature of a layout design problem.

The present study explored the use of TOPSIS and fuzzy TOPSIS in solving a layout design problem. A practical application from an IC packaging company was adopted for empirical testing. This study aimed at searching an improved solution to an existing design. Moreover, the methods and experiences learned from the study can be valuable to the company's future strategic planning. Empirical results showed that the proposed methods are viable approaches in solving the proposed layout design problem. TOPSIS is a viable method for the proposed problem and is suitable for the use of precise performance ratings. When the performance ratings are vague and inaccurate, then the fuzzy TOPSIS is the preferred technique.

Each layout design application is unique in nature, i.e., there are different attributes associated with different applications; thus, the success of the present study has no guarantee for its applicability to other applications. Judicious use of a design method is advised in solving a specific application. In addition, there exists other worth investigating MADM methods for a layout design problem. This becomes one of the future research opportunities in this classical yet important research area.

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