

Developing a heuristic algorithm for order production planning using network models under uncertainty conditions

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Abstract

In this paper, a practical and appropriate algorithm is provided for order production planning under uncertainty conditions. Producing the order production projects using different methodologies is facing uncertainty and therefore specific techniques are required to determine their production planning schedule. Network models combining fuzzy logic to define the sequence of the production stages and uncertainty are appropriate tools for order production planning modeling. In this paper, start time parameters of each activity, duration of each activity and number of loops in the products production network are defined as triangular fuzzy numbers (pessimistic, possible, optimistic) and results of the production planning are defined as fuzzy sets. In this paper, network calculations including fuzzy forward CPM are performed. Output of this algorithm is a fuzzy order production schedule. This method is more practical than existing production planning schedules determination methods and has less calculation and has upgrade and mechanization capability.

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1. Introduction

Order production planning is one of the major columns of production project management, products evaluation and selection. Inappropriate and unreal production planning of the order production projects leads to errors in current budget cost assessment, resource planning and provision, imposes penalties on contracts and disparity in project progress reports and etc. One of the major reasons for inappropriate and unreal planning of the order production projects is disuse of suitable techniques for production planning of these projects. Production planning of the order production projects has specific features which distinct them from other projects such as uncertainty in definition, project activities duration and sequence, numerous cycles and resilience and non-recurrence of the project activities which require special techniques to determine the production planning schedules for these products. Since, different approaches including step-by-step, waterfall, prototype, spiral, rapid and parallel methods have been provided for order projects production planning. In projects order

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production planning, for all models we use deterministic networks techniques like CPM and rarely the stochastic networks.

In deterministic networks techniques like CPM, all of the order production network parameters (definition of activities, sequence of activities and duration of activities) are defined deterministically. These techniques are simple and effective and also several order products such as MSP, Primavera and Timeline have been developed to increase their application. But generally, these techniques are not able to show the cycles, return loops and uncertainty of the activities time and definition in order products projects. Therefore, results of these techniques are widely inaccurate.

In stochastic networks techniques, parameters like definition and sequence of the activities, duration of activities and their relationships are stochastic variables. Cyclic and acyclic stochastic networks are the prominent examples of these networks [1]. In acyclic stochastic networks, only the activity duration parameter is uncertain and is estimated by probability distribution. This method in real-life problems gives much more real results than certain method (CPM) and numerous papers have been provided to extend this approach [5]. But these methods are not able to consider the uncertainty of the activities duration and definition and also cycles in the order projects production planning.

Cyclic stochastic networks using logical nodes, stochastic paths and recurrent loops are suitable tools for order production projects modeling and they are able to solve the activities definition, duration and loops uncertainty problem in these projects. But lack of the required statistics and information for stochastic parameters estimation in order production projects and lack of the analytical methods in cyclic stochastic networks by combination of the various production stages is a difficulty toward the application of this method to determine the order production planning schedules. Hence, to solve the cyclic networks problems generally we use the simulation method and combination of the various production stages using computer programs [2,3]. In this paper we try to develop a practical and suitable technique for order production planning schedule. In this case, order production planning is performed in a more real way.

2. Production planning techniques under uncertainty conditions

Fuzzy logic is a suitable tool to define the uncertainty and ambiguity in human performance assessments. In order production planning determination techniques, parameters and their relationships are fuzzy and fuzzy sets are used to define the uncertainty of these parameters. Main reasons for the application of fuzzy logic in order production planning are

1. ambiguity and uncertainty in definition and sequence of activities,
2. uncertainty in activity duration assessment,
3. inaccessibility and lack of information of past and similar projects,
4. less information requirement of the fuzzy approaches in comparison with stochastic methods,
5. facility of fuzzy methods solution and less calculation in comparison with stochastic methods.

Chanas and Kamberovsky [2] in 1981 and Henry Pod [3] in 1979 first applied fuzzy logic in order production planning. They replaced the stochastic parameters with fuzzy parameters in cyclic stochastic network. Since several techniques have been developed for fuzzy project scheduling which from the application point of view can be divided to three categories of fuzzy time, fuzzy network and cyclic fuzzy network [4–6].

In the network technique with fuzzy time, the only fuzzy parameter is the network time and other parameters like operations definition and operations sequence are certain. Many articles have been issued in this context and so many approaches have been extended [8–10]. These techniques give much more real results in comparison with deterministic techniques but they are not able to model the uncertainty of the operations definition and sequence and also the cycles in order production planning schedule determination.

In the network technique with fuzzy parameters, operations definition, duration and sequence parameters can be defined as fuzzy parameters but cycles can not be considered by these techniques and also no suitable solution method has been provided except Evans et al. [7] solution method provided in 1989.

Cyclic fuzzy networks and cyclic stochastic networks are the same with this exception that stochastic parameters are replaced with fuzzy parameters. These networks using the high capabilities of the cyclic

stochastic networks and fuzzy sets provide suitable tools for order production planning schedule determination. Cyclic fuzzy networks were first introduced by Inakora and Nishikara [8] in 1984 and later Cheng [9] developed a specific state of the cyclic fuzzy networks which cannot be used for fuzzy production planning schedules. The approach developed by Inakora has investigated the various node combinations in cyclic fuzzy networks and can be used for order production planning schedule determination. But unfortunately, it is not a practical method and its computational logic is confronted with some difficulties as below:

1. In this method, activity duration is considered as discrete fuzzy numbers which is impractical in reality and makes the data acquisition process very difficult.
2. This algorithm is based on the stages, paths and loops determination which is very difficult in the case of large networks and makes the computations more complex.
3. Loops frequency determines the level of computational complexity since recurrent loop paths increases the computations.
4. This method is suitable for computerization of computations and along with upgrading the industrial network.
5. Existing method's computations are based on paths and use less job stages.

Hapke et al. [10] and Hapke and Slowinski [11] were the first to study fuzzy models for projects production planning with limited resources. They extended the priority rule according to sequential and parallel production planning plans to deal with fuzzy parameters. Hapke and Slowinski [12] discuss the application of simulated annealing for solving the multi-objective fuzzy resource-constrained production planning problem. The procedure is the adaptation of Pareto simulated annealing procedure developed by Czyzak and Jaskiewicz [13].

Özdamar and Alanaya [14] study software development projects and offer a non-linear mixed-binary mathematical problem formulation and accompanying solution heuristics. Their model incorporates uncertainties related to activity durations and network topology. Activities may be performed in one of different methods with a corresponding fuzzy duration. The objective function is to minimize the project duration. Wang [15] has developed a fuzzy set approach for order production planning having imprecise temporal information. The project has a fuzzy ready time and fuzzy deadline and the activities are assumed to have a fuzzy duration, all described by trapezoidal fuzzy numbers. The objective is to determine a start time for each activity such that the fuzzy ready time, deadline, precedence and resource constraints are satisfied.

Wang [16] has presented a fuzzy beam search approach for solving the problem under the objective of minimizing the production planning risk. Wang [17] describes a genetic algorithm for solving the problem under the objective of maximizing the worst case order production planning performance.

According to the existing methods problems and insufficiencies, a new method has been developed in this paper with computerization and upgrading capabilities and using more job stages with the aim of making this method more practical. Activity duration and loops repeat number parameters in cyclic fuzzy networks are represented by triangle fuzzy numbers (pessimistic, probable and optimistic) and the output activities of groups belong to fuzzy sets. In this method, required calculations like CPM method are based on the order production stages. Output of this algorithm is a production planning including all of the order production stages which are represented by fuzzy numbers. The validity and practicality of this method on the existing stochastic and fuzzy networks has been implemented and compared using several examples. Also, finally this method has been implemented on several real-life order production projects and the provided technique has been modified during the application.

3. Development of a solution method for cyclic fuzzy networks

The first stage of the production planning schedule determination for order production is the definition of nodes, paths and loops required for order production system. As much as we get to know the problem, the result of the schedule will be more real and this is a part of the project management art. In this investigation, cyclic fuzzy networks are used to model the software projects and network parameters are represented by fuzzy sets. In this method first we evaluate the projects information according to definitions and assumptions below and project job network is described. The next step is solving the cyclic fuzzy network with regard of the

existing techniques insufficiencies; a new algorithm has been developed for this objective. The output of this algorithm includes the job process stages schedule and project completion time as fuzzy numbers.

3.1. Definition of a cyclic fuzzy network

Cyclic fuzzy network and cyclic stochastic network are the same except that in cyclic stochastic network; stochastic parameters are replaced with fuzzy parameters which include job process stages, paths and loops.

3.1.1. Project process stages

Various models are available for order production. Among these models, step-by-step, waterfall, parallel, prototype, components collection can be mentioned. In this paper a multi-stage production model has been studied for order production but the extended method is easily applicable for other order production models.

3.1.2. Directional fuzzy paths

In this paper, stochastic paths have been replaced by fuzzy paths. Each fuzzy path starts from one node and ends to another node. Each path is specified with two parameters:

- t_{ij} : Fuzzy time of the activities process indicates the approximate time of the project activities process.
- μ_{ij} : Membership degree of the path which indicates the feasibility of that path.

3.1.3. Cycles and loops

Each L_{ni} loop consists of one or more activity which is executable more than once. Fig. 1 shows each loop with two parameters:

- μ_{Lni} = loop occurrence possibility,
- r_{Lni} = loop frequency.

3.2. Network assumptions

In this algorithm, for practicality and ease of calculations, the following assumptions are considered. Indeed, this solution method can easily be generalized to the general state and without the following assumptions.

1. *Activities duration* (t_{ij}). These times are represented as linguistic variables. In this project, for practicality and ease of the calculations, we have used triangular fuzzy numbers (pessimistic, probable, optimistic) for the estimations [8]. In general, expert people's ideas and opinions can be useful to determine each activity's duration. Of course, expert people's ideas may differ. For the expert people's ideas to be convergent, fuzzy Delphi approach can be used [9]. A more simple method is using the average of people's ideas in formula (5). In this paper, we calculate the average of the fuzzy numbers and use them to determine each activity's fuzzy duration.

Fig. 2 shows a situation when activity duration is in (t_1-t_3) but most of the times it is equal to t_2 .

2. *Loop frequency* (r_{Lni}). The loop frequency also is a fuzzy number which can be observed approximately for five times (maximum 8 times, minimum 3 times). To display the fuzzy number of the loops, we can act like activities fuzzy duration determination except that frequency number unit is used instead of the time unit.

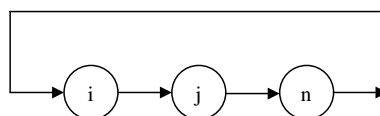


Fig. 1. Loop models network.

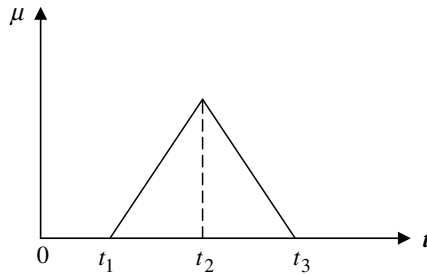


Fig. 2. Triangular representation of the fuzzy activities duration.

3. *Loops membership degrees.* In this project, according to model’s assumptions, a definitive number between 0 and 1 is used to assess membership degree of the loops. In practice, possibility of the loops can be stated approximately (approximately 20%, between 20% and 30% and etc.). These membership degrees are stated by project administrators which are fuzzy concepts and can be represented as fuzzy numbers. The closer these numbers to 1, it means that activity and loop occurrence is more possible (membership degree of the loop is higher). If degree of membership is described as fuzzy numbers, in this method we use the average of the triangular fuzzy numbers in relation (5).

4. *Loops occurrence feasibility* is equal for all frequencies and number of the activities on the path of the loop is totally definite.

5. If $M = (a_1, b_1, c_1)$ and $N = (a_2, b_2, c_2)$ represent two triangular fuzzy numbers then, required fuzzy calculations are performed as below:

$$\text{Fuzzy addition : } M \oplus N = (a_1 + a_2, b_1 + b_2, c_1 + c_2). \tag{1}$$

$$\text{Fuzzy multiplication : } M \otimes N = (a_1 \times a_2, b_1 \times b_2, c_1 \times c_2). \tag{2}$$

$$\text{Fuzzy and a natural number multiplication : } r \otimes M = (r \cdot a, r \cdot b, r \cdot c). \tag{3}$$

$$\text{Fuzzy maximum function : } \max(N, M) = [(a_1, a_2) \vee (b_1, b_2) \vee (c_1, c_2)]. \tag{4}$$

$$\text{Triangular fuzzy numbers average : } m(A) = \left[\frac{\sum_{i=1}^n a_i}{n}, \frac{\sum_{i=1}^n b_i}{n}, \frac{\sum_{i=1}^n c_i}{n} \right]. \tag{5}$$

$$\text{Fuzzy minimum function : } \min(N, M) = [(a_1, a_2) \wedge (b_1, b_2) \wedge (c_1, c_2)]. \tag{6}$$

3.3. Network solution method

Concerning the difficulties and insufficiencies of the existing methods and with the aim of practicality and computerization and upgrading capabilities of fuzzy networks, in this paper a new method have been developed to solve fuzzy networks. In this algorithm, network calculations are similar to the fuzzy CPM method. General stages of these networks are described below:

1. Network activities are evaluated and average activity duration is calculated for each activity.
2. Set release time of the start node to zero:

$$T_{\text{start}} = ST_{\text{start}} = (0, 0, 0).$$

3. Calculate and evaluate the average release time of each node from beginning to end.
4. By evaluating the final node, project’s completion time is obtained and the network is scheduled:

$$T_{\text{Project}} = AT_{\text{end}}.$$

5. Analyze the order production planning results.

4. Network activities evaluation

Existent activities in cyclic fuzzy networks may be nested in several loops (Fig. 3) and iteration possibility of the inner loops is independent of the outer loops which must be considered during average duration activity. In this case, time value of each activity primarily includes the time of the activity plus the average value time of that activity in recurrence of the loops. Accordingly, value of each activity is calculated as below.

4.1. Calculating the unit time value for an activity (t_{jk})

Unit time value of an activity is the average fuzzy time of an activity obtained through the expert people’s consultation.

4.2. Calculating the activity frequency resulted from recurrent loops

Since loops frequency is a fuzzy number, thus each activity’s frequency is obtained from the sum of the products of loops fuzzy numbers and their occurrence possibility:

$$N_{jk} = \mu_{Lkj} \otimes r_{Lkj} + \mu_{Lmj} \otimes r_{Lmj} + \mu_{Lnj} \otimes r_{Lnj}. \tag{7}$$

4.3. Calculating the average activity duration

Each activity is obtained through the sum of the activity time and product of activity frequency and activity time:

$$t'_{jk} = t_{jk} + N_{jk} \otimes t_{jk}. \tag{8}$$

5. Nodes evaluation

Nodes evaluation is performed according to input and output activities. Primary release time for a node (ST_i) is equal to the maximum finishing time of the all input activities to that node. This time is obtained according to the completion of the input activities to each node. After the first evaluation of the nodes, if there is a loop in output paths (i.e., there is a return possibility) then time of the loop is added to the beginning released time of the node. Therefore; we define another parameter for the node calling it the average released time of the node (MT_n). This time indicates the average of the possible released times of the node. Evaluation procedure for each node is as below.

5.1. Calculating input activities finish time

Each activity’s finish time is equal to sum of the average release time of the first node and fuzzy activity duration. Therefore, this finish time is calculated as below regardless of the input activities:

$$ft_{i-n} = MT_i \oplus t_{i-n}. \tag{9}$$

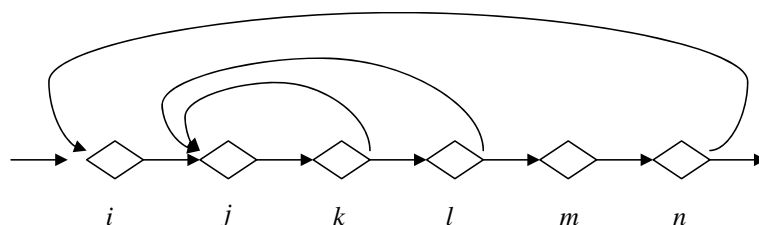


Fig. 3. Fuzzy network loops representation.

5.2. Calculating the node primary released time (ST_i)

Amount of this parameter (ST_i) is calculated according to the input activities finish time. According to order production model’s definition, a node is released when all of its input activities are completed. Thus, primary release time of each node can be equal to the maximum finish time of the input activities. Therefore, generally the primary release time of a node can be envisioned as average of the input activities finish time and activity duration in recurrent loops. As an example, for node n in Fig. 3 we will have

$$ST_n = \max(ft'_{mn}, ft'_{ln}). \tag{10}$$

5.3. Analysis of the production planning results

Outputs of this method include production planning schedules for each stage of the order production which can be obtained through each node and activities evaluation calculations. Since, calculations are based on the nodes, it is similar to forward CPM calculations and has the capability to be computerized and upgraded. Project completion time is a fuzzy number. Since our input parameters are triangular fuzzy numbers, then also the project completion time will be a triangular fuzzy number. It means that in fuzzy networks, review of the project completion time to a definitive number will be a fuzzy number (interval) which is consistent with reality and eliminates the stresses and tensions in project control sessions and also cost assessment process will be more real.

If we assume the project completion time to be a triangular fuzzy number (a, b, c) then this interval indicates that the day is completed at this interval with α confidence level. α is considered as a risk level and decision maker can calculate and analyze the project completion time interval at various risk levels (Fig. 4). Average of the project completion time can be obtained through defuzzification in formula (5).

6. Validity and reliability of the extended algorithm

To test the new method and evaluate the validity and reliability of the extended model, we must make comparison between this method and existing methods. For this purpose, we have selected and designed several theory examples and solved them using new and existing methods and compared the results. This method has been applied to several practical projects at ICT organization of municipality of Tehran. We refer to one of them as an example.

In this section, an example of the cyclic probabilistic networks in Ref. [2] has been considered. This example has been solved by new and existing methods. As a result, project completion time in stochastic state (normal distribution) is equal to 12.5 but fuzzy project completion time is (11.5, 14.7, 17.5) average fuzzy project completion time using formula 8 is equal to 14.7 considered project is completed during 18 months. As you can see, error of the fuzzy method is equal to 3.3 months which is less than the stochastic method (5.5 months). Another example of the cyclic fuzzy networks is extracted from Ref. [12] and has been solved by cyclic fuzzy network solution and extended methods. Average of the fuzzy project completion times is 9.5 and 16, respectively. Delay time by this method regarding the real project completion time is equal to 2 months and 8.5 months. For both of the examples above, it is evident that fuzzy extended method’s solution has smaller error values.

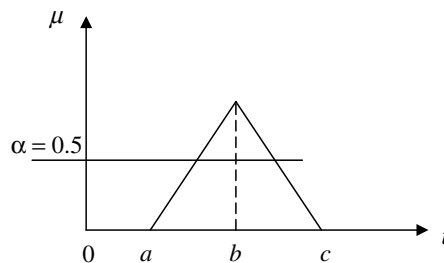


Fig. 4. Fuzzy project completion time diagram.

For more assurance of the validity and reliability of the extended algorithm, an order production project has been planned using CPM method. Procedure is as below.

6.1. Project network definition and drawing (description)

This project encompasses designing, production and implementation of the order production. Cyclic fuzzy network of this project is depicted in Fig. 5. Project’s input parameters are listed in Table 1.

6.2. Solving cyclic networks

This stage includes network activities evaluation using formulae (7) and (8) and evaluation of the nodes using formulae (9) and (10). Results are provided in Tables 2 and 3.

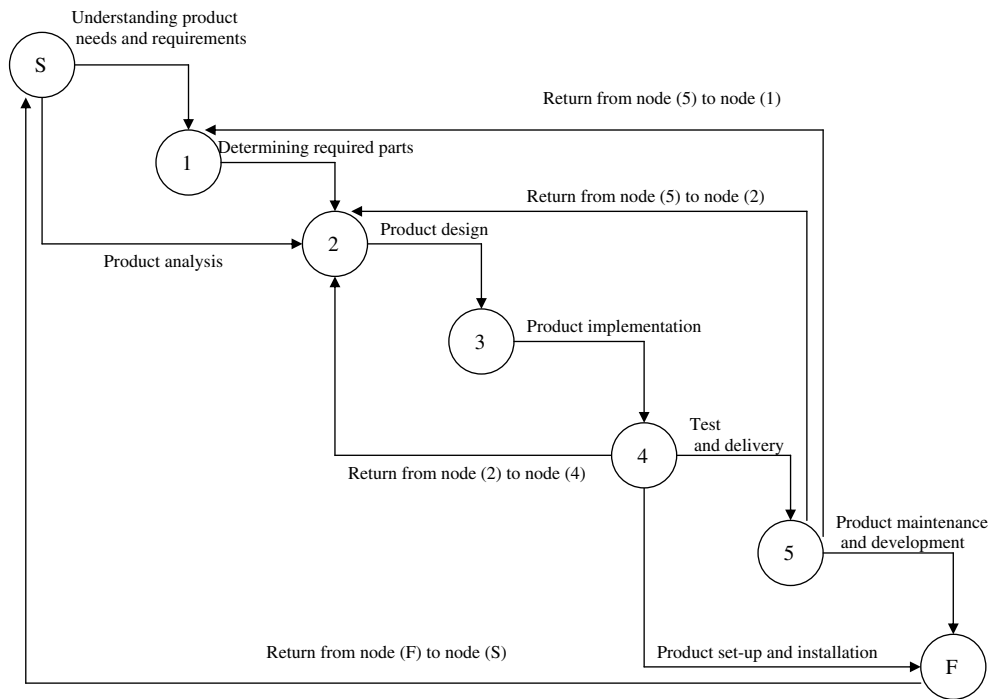


Fig. 5. Order production model.

Table 1
Order production network loops and activities

Rank	Activity code (loop)	Activity description (loop)	Activity duration (day), loop frequency (number)	Activity occurrence probability (loop)
1	S-1	Understanding product needs and requirements	(5, 7, 10)	1
2	1-2	Determining required products	(11, 14, 20)	1
3	S-2	Product analysis	(20, 28, 35)	1
4	2-3	Product design	(17, 20, 25)	1
5	3-4	Product implementation	(28, 30, 36)	1
6	4-5	Product test and delivery	(5, 8, 11)	1
7	5-F	Product maintenance and expansion	(8, 11, 15)	1
8	4-F	Product set-up and installation	(5, 8, 12)	1
9	4-2	Loop (4 → 2)	(1, 2, 3)	0.2
10	5-1	Loop (1 → 5)	(3, 4, 5)	0.4
11	5-2	Loop (5 → 2)	(2, 3, 4)	0.3
12	F-S	Loop (F → S)	(2, 3, 4)	0.3

Table 2
Activities duration including iteration loops

Activity	Iteration loops	Loop occurrence probability	Activity duration	Activity frequency	Frequency (loop)	Final activity completion time	Final activity frequency
S-1	(F-S)	0.3	(5, 7, 10)	(0.6, 0.9, 1.2)	(2, 3, 4)	(8, 13, 22)	(1.6, 1.9, 2.2)
1-2	(F-S)	0.3	(11, 14, 20)	(1.2, 1.8, 2.4)	(2, 3, 4)	(24, 39, 68)	(2.2, 2.8, 3.4)
	(5-1)	0.4			(3, 4, 5)		
S-2	(F-S)	0.3	(20, 28, 35)	(0.6, 0.9, 1.2)	(2, 3, 4)	(32, 53, 77)	(1.6, 1.9, 2.2)
2-3	4-2	0.2	(17, 20, 25)	(2.6, 3.8, 5)	(1, 2, 3)	(61, 96, 150)	(3.6, 4.8, 6)
	5-1	0.4			(3, 4, 5)		
	5-2	0.3			(2, 3, 4)		
	F-S	0.3			(2, 3, 4)		
3-4	4-2	0.2	(285, 30, 36)	(2.6, 3.8, 5)	(1, 2, 3)	(101, 144, 216)	(3.6, 4.8, 6)
	5-1	0.4			(3, 4, 5)		
	5-2	0.3			(2, 3, 4)		
	F-S	0.3			(2, 3, 4)		
4-5	5-1	0.4	(5, 8, 11)	(2.4, 3.4, 4.4)	(3, 4, 5)	(11, 22, 37)	(2.2, 2.8, 3.4)
	5-2	0.3			(2, 3, 4)		
	F-S	0.3			(2, 3, 4)		
5-F	F-S	0.3	(8, 11, 15)	(0.6, 0.9, 1.2)	(2, 3, 4)	(13, 21, 33)	(1.6, 1.9, 2.2)
4-F	F-S	0.3	(5, 8, 12)	(0.6, 0.9, 1.2)	(2, 3, 4)	(8, 15, 26)	(1.6, 1.9, 2.2)

Table 3
Order production planning schedule

Node	Input activity	Final activity completion time	Input activities finish time	Node release time
S	–	(0, 0, 0)	(0, 0, 0)	(0, 0, 0)
1	S-1	(8, 22, 3)	(8, 13, 22)	(8, 13, 22)
2	1-2	(24, 39, 68)	(32, 52, 90)	(40, 68, 99)
	S-2	(32, 53, 77)	(40, 68, 99)	
3	2-3	(61, 96, 150)	(101, 164, 249)	(101, 164, 249)
4	3-4	(101, 144, 216)	(202, 308, 465)	(202, 308, 465)
5	4-5	(11, 22, 37)	(213, 330, 502)	(213, 330, 502)
F	5-F	(13, 21, 33)	(226, 351, 535)	(226, 351, 535)
	4-F	(8, 15, 26)	(226, 345, 528)	

6.3. Project completion time calculation and final results analysis

Since the project is completed, results of these two methods can be compared with the real project completion time. Project completion time estimation by CPM method is equal to 3 months which in comparison with the real project completion time (10 months) has much more error. Result of the project completion time estimation by new fuzzy method is equal to (7.5, 11.7, 17.8) months which is closer to the real project completion time and if we only consider the average of the fuzzy project completion time (12 months), in comparison with CPM method has much less error.

7. Production planning diagram and schedules

Production stages for each product accompanying production recurrent cycles are provided in Fig. 5. This diagram consisted of five activities and three recurrent loops. This diagram is represented as activity table and requirement loops in (1). Activity durations with calculated recurrent loops are presented in Table 2. Finally, Table 3 provides an order production planning schedule.

8. Conclusion

Fuzzy cyclic networks using the high capabilities of the network models and fuzzy sets are suitable tools for research projects planning. These networks and stochastic networks are the same in which the stochastic parameters are replaced with fuzzy parameters which include logical nodes, fuzzy branches and loops. In this paper, a new method is developed for solving the cyclic fuzzy networks. In this method, input parameters include project activities network, determining the activities fuzzy time parameter, membership degree for each activity, fuzzy frequency of the loops and loops occurrence possibility. For this purpose, we use simple rules of addition, multiplication, maximum and minimum. Output of this method includes a fuzzy schedule for the project production stages.

To evaluate the validity and reliability of this method, it has been applied to theory examples of the cyclic stochastic and cyclic fuzzy networks which have given much better results. Also this method has been applied to a real order production project and the obtained results were closer to reality in comparison with other methods. Advantages of this method over deterministic and stochastic networks techniques are as below:

1. Cycle fuzzy network techniques like cyclic stochastic techniques have more capability to display the possible activities during projects, cycles and resiliences planning especially in comparison with deterministic networks which has less variability during the implementation.
2. In this technique, output parameters like project completion time are fuzzy numbers which are more real. Furthermore, this technique reduces the stresses and strains in control project sessions and progress reports are closer to reality.
3. Less information requirement to estimate the parameters in comparison with stochastic methods and parameters estimation using fuzzy logic is closer to reality.
4. Facility of the fuzzy calculations over stochastic calculations.
5. Capability to computerize and upgrade the fuzzy methods toward stochastic methods and much more real-life application.

Use of fuzzy numbers in activities and loops occurrence possibility estimation, activities duration, sensitivity analysis, upgrading activities network, cost assessment and achievement rate estimation are among the issues to be mentioned to extend this method.

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