A Fuzzy Model for Exploiting Quality Function Deployment

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Abstract—Quality function deployment (QFD) is the product development process to maximize customer satisfaction. The engineering design characteristics related to product performance are specified for this purpose. For dealing with the fuzzy nature in the product design processes, fuzzy approaches are applied to represent the relationships between customer requirements (CRs) and engineering design requirements (DRs) as well as among the DRs. A new measure for evaluating the fuzzy normalized relationships is derived. A fuzzy model is formulated to determine the fulfillment level of each DR for maximizing the customer satisfaction under the resource limitation and the considerations of technical difficulty and market competition. The producing ranges of fulfillment level of each DR and those of customer satisfaction can provide the QFD team with more information.

An example is used to illustrate the model. © 2003 Elsevier Ltd. All rights reserved.

Keywords—Quality function deployment (QFD), Fuzzy numbers, Fuzzy linear programming.

1. INTRODUCTION

Quality function deployment (QFD) originated in Japan in the 1970s and became increasingly popular in the western world in the 1980s. It has been successfully applied in many Japanese organizations to improve processes and to build competitive advantages [1]. Today, companies are successfully using QFD as a powerful tool that addresses strategic and operational decisions in businesses [2,3]. Quality function deployment is a systematic approach for ensuring that customers' voices are deployed in the product planning and design stages. The desires of customers on a product are taken into account, through conducting a survey by the marketing department and are treated as a set of customer requirements (CRs). A number of engineering design requirements (DRs) that affect CRs are also identified to maximize customer satisfaction. In general, a QFD team is organized to determine the improvement levels of DRs by analyzing the relationships between CRs and DRs as well as among the DRs, considering the cost and other organizational constraints.

However, QFD still has several limits in applications and therefore can be improved further. QFD team members usually subjectively determine the relationships between CRs and DRs and among the DRs based on past experience, due to the lack of precise information from customer requirements. Moreover, information for product design is often limited and imprecise, particu-
larly when developing an entirely new product, such that engineers usually do not have complete knowledge about the impacts of engineering characteristics on customer requirements (CRs). These considerations have made the applications of some studies using crisp data restricted, such as Park and Kim [4] and Trappey et al. [5]. Instead, some researchers presented computer-aided systems to aid engineers in designing engineering factors. Fung et al. [6] proposed a fuzzy customer requirement inference system in which the product attributes could be mapped out. Moskowitz and Kim [7] provided a decision support system for optimizing product designs. Tempioni et al. [8] developed a reasoning scheme for inferring requirement relationships between CRs and DRs, as well as among the DRs. Nevertheless, the development of these systems usually requires professional knowledge and experiences to establish rules and facts in ensuring that the systems can work well.

Some models are also formulated for determining the levels of engineering design requirements based on fuzzy set theory. Kim et al. [9] proposed a fuzzy theoretical modeling approach to QFD by formulating fuzzy multiobjective models, assuming that the function relationships between CRs and DRs and that among the DRs can be identified using benchmarking data of customer competitive analysis. But it would be difficult, particularly when developing an entirely new product. Some researchers ever developed fuzzy approaches to address complex and often imprecise problems in customer requirement management by applying fuzzy sets, fuzzy arithmetic, and/or fuzzy defuzzification techniques [10-13]. However, the interrelationships among the engineering characteristics were not properly incorporated in these models.

Instead of the fuzzy approaches mentioned above, this study considers not only the inherent fuzziness in the relationships between CRs and DRs, but also those among DRs. Two kinds of fuzzy relationships are aggregated, based on Wasserman's study [14], to obtain the fuzzy normalized relationship matrix in which each cell is represented by a fuzzy number. Unlike the existing approaches [15], we proposed new expressions for those fuzzy numbers to obtain more shortened $\alpha$-cuts, such that fuzzy technical importance ratings for engineering design requirements can be determined in terms of $\alpha$-cuts with less uncertainty. Under the resource limitation and the considerations of technical difficulty, and market competition, we then formulated a fuzzy LP model to determine the optimal fulfillment degrees of DRs at each $\alpha$-cut for achieving the optimal customers' satisfaction. An illustrative example is used to demonstrate the feasibility of the proposed approach.

In the following section, a fuzzy normalized relationship matrix of QFD is introduced, and new expressions are derived for the $\alpha$-cuts of each fuzzy relationship value. Fuzzy technical importance rating for each engineering design requirement at each $\alpha$-cut is then determined. Section 3 formulates the QFD planning problem as a fuzzy model to determine the fulfillment levels of DRs to produce the maximum customer satisfaction. An example is given to demonstrate our approach in Section 4. Finally, conclusions are provided in Section 5.

2. FUZZY RELATIONSHIP MATRIX

The relationships between CRs and DRs in QFD are represented in the matrix form, which is also called the house of quality (HOQ), as shown in Figure 1. The matrix has two dimensions, i.e., customer wants and engineering design requirements. A triangular-shaped matrix placed over the engineering design requirements corresponds to the correlations between them. A point system is applied to evaluate the relationships between CRs and DRs, as well as among the DRs based on the strength of relationship in the existing approaches [14]. In the figure, $R_{ij}$ denotes the score of the relationship between the $i^{th}$ CR and the $j^{th}$ DR, and $r_{jn}$ is the correlation score for the $j^{th}$ and $n^{th}$ DRs.

For obtaining relative relationship degrees of DRs with respect to some CR and dealing with the dependence among DRs, Wasserman [14] proposed a normalized transform on the relationship
values contained in the relationship matrix, expressed as equation (1):

$$R'_{ij} = \frac{\sum_{k=1}^{n} R_{ik} \gamma_{kj}}{\sum_{j=1}^{n} \sum_{k=1}^{n} R_{ik} \gamma_{kj}}$$

where $R'_{ij}$ is the normalized relationship value between customer requirement $i$ and engineering design requirement $j$, $i = 1, 2, \ldots, m$, $j = 1, 2, \ldots, n$, and $\sum_{j} R'_{ij} = 1$ for each $i$. The relationships are quantified through the use of a 1-3-9 or 1-5-9 scale to denote weak, medium, and strong relationship degrees between CRs and DRs and the correlation among DRs. Although the relationships are imprecise in nature, the crisp point system is used to evaluate the degree of relationship in the above approach. For representing the relationships meaningfully, fuzzy numbers are used in this study to assess the relationship in an ambiguous manner. Therefore, $\tilde{R}_{ik}$ and $\tilde{\gamma}_{kj}$ are the fuzzy numbers representing the fuzzified quantitative relationship degrees between CRs and DRs and the correlation among DRs, respectively. The corresponding fuzzy normalized relationship can be formulated as

$$\tilde{R}'_{ij} = \frac{\sum_{k=1}^{n} \tilde{R}_{ik} \tilde{\gamma}_{ki}}{\sum_{j=1}^{n} \sum_{k=1}^{n} \tilde{R}_{ik} \tilde{\gamma}_{kj}}$$

Without loss of generality, the two kinds of fuzzy numbers, $\tilde{R}_{ik}$ and $\tilde{\gamma}_{kj}$, are defined on $[0, 1]$ in this study, such that $\tilde{R}'_{ij}$ will also be in the same interval. In order to define the membership function of the fuzzy normalized relationship $\tilde{R}'_{ij}$, $\alpha$-cuts and the extension principle [16,17] are used, since $\tilde{R}'_{ij}$ is the function of $\tilde{R}_{ik}$ and $\tilde{\gamma}_{kj}$. First, $\tilde{R}_{ik}$ and $\tilde{\gamma}_{kj}$ can be represented by different levels of $\alpha$-cuts expressed as

$$(R'_{ik})_{\alpha} = \min \left\{ R_{ik} \in R_{ik} \mid \mu_{\tilde{R}_{ik}} (R_{ik}) \geq \alpha \right\}, \max \left\{ R_{ik} \in R_{ik} \mid \mu_{\tilde{R}_{ik}} (R_{ik}) \geq \alpha \right\}$$

as $\alpha$, $\alpha$-cuts of $R'_{ik}$.
where \( \alpha \in [0,1] \) and \( \mu_{R_{ik}}(R_{ik}) \in [0,1] \) and \( \mu_{r_{kj}}(r_{kj}) \in [0,1] \) are the membership degrees of \( R_{ik} \) and \( r_{kj} \), respectively. In addition, based on the extension principle, the membership degree of fuzzy normalized relationship \( \tilde{R}'_{ij} \) can be defined as

\[
\mu_{\tilde{R}'_{ij}}(R'_{ij}) = \sup_{{R,r}} \left\{ \mu_{\tilde{R}_{ik}}(R_{ik}), \mu_{r_{kj}}(r_{kj}), \forall k,j \right\} \left( R'_{ij} = \frac{\sum_{k=1}^{n} R_{ik} r_{kj}}{\sum_{j=1}^{n} \sum_{k=1}^{n} R_{ik} r_{kj}} \right).
\]

The membership function of \( \tilde{R}'_{ij} \) can be constructed by deriving the \( \alpha \)-cuts of \( \tilde{R}'_{ij} \). Based on equation (4), the lower and upper bounds of \( \alpha \)-cuts of \( \tilde{R}'_{ij} \) can be solved, using Kao and Liu's idea [15], as

\[
(\tilde{R}'_{ij})_L^\alpha = \min R'_{ij} = \frac{\sum_{k=1}^{n} R_{ik} r_{kj}}{\sum_{j=1}^{n} \sum_{k=1}^{n} R_{ik} r_{kj}},
\]

s.t. \( (\tilde{R}_{ik})_L^\alpha \leq R_{ik} \leq (\tilde{R}_{ik})_U^\alpha, \quad \forall k, \)

\[
(\gamma_{kj})_L^\alpha \leq r_{kj} \leq (\gamma_{kj})_U^\alpha, \quad \forall k,j,
\]

\[
(\tilde{R}'_{ij})_U^\alpha = \max R'_{ij} = \frac{\sum_{k=1}^{n} R_{ik} r_{kj}}{\sum_{j=1}^{n} \sum_{k=1}^{n} R_{ik} r_{kj}},
\]

s.t. \( (\tilde{R}_{ik})_L^\alpha \leq R_{ik} \leq (\tilde{R}_{ik})_U^\alpha, \quad \forall k, \)

\[
(\gamma_{kj})_L^\alpha \leq r_{kj} \leq (\gamma_{kj})_U^\alpha, \quad \forall k,j.
\]

From the above formulations, the lower and upper bounds of \( \alpha \)-cuts of \( \tilde{R}'_{ij} \), \( (\tilde{R}'_{ij})_L^\alpha \), and \( (\tilde{R}'_{ij})_U^\alpha \) can be obtained by the following simplified expressions as

\[
(\tilde{R}'_{ij})_L^\alpha = \frac{\sum_{k=1}^{n} (\tilde{R}_{ik})_L^\alpha (\gamma_{kj})_L^\alpha}{\sum_{j=1}^{n} \sum_{k=1}^{n} (\tilde{R}_{ik})_L^\alpha (\gamma_{kj})_L^\alpha},
\]

\[
(\tilde{R}'_{ij})_U^\alpha = \frac{\sum_{k=1}^{n} (\tilde{R}_{ik})_U^\alpha (\gamma_{kj})_U^\alpha}{\sum_{j=1}^{n} \sum_{k=1}^{n} (\tilde{R}_{ik})_U^\alpha (\gamma_{kj})_U^\alpha}.
\]

The solutions from equation (6) are the possible extreme ranges at each \( \alpha \)-cut. Therefore, these ranges may be greater than the actual necessary ones, so as to produce unnecessary impreciseness. For dealing with this problem, the present study proposes the modified expressions to improve the outcomes. The modified ones are derived as follows. Referring to equation (1), the normalized relationship value \( R'_{ij} \) is defined as

\[
R'_{ij} = \frac{\sum_{k=1}^{n} R_{ik} r_{kj}}{\sum_{j=1}^{n} \sum_{k=1}^{n} R_{ik} r_{kj}} = \frac{\sum_{k=1}^{n} R_{ik} r_{kj}}{\sum_{i=1}^{n} \sum_{k=1}^{n} R_{ik} r_{ki} + \sum_{k=1}^{n} R_{ik} r_{kj}}.
\]

\[
R'_{ij} = \frac{\sum_{k=1}^{n} R_{ik} r_{kj}}{\sum_{j=1}^{n} \sum_{k=1}^{n} R_{ik} r_{kj}}.
\]
where

\[ 0 \leq (\mathcal{R}_{ik})_\alpha^L \leq R_{ik} < (\mathcal{R}_{ik})_\alpha^U \leq 1, \quad \forall k, i = 1, \ldots, m, \]
\[ 0 \leq (\gamma_{kj})_\alpha^L \leq r_{kj} \leq (\gamma_{kj})_\alpha^U \leq 1, \quad \forall k, j. \]

Let \( \phi = \sum_{k=1}^{n} R_{ik} r_{kj} \) and \( \varphi = \sum_{l \neq j}^{n} \sum_{k=1}^{n} R_{ik} r_{kl} \).
Then equation (7) is expressed as

\[ f(\phi) = \frac{\phi}{\varphi + \phi}. \]

Since

\[ f'(\phi) = \frac{\varphi}{(\varphi + \phi)^2} \geq 0, \]

\( f(\phi) \) is an increasing function and

\[ \sum_{k} (\mathcal{R}_{ik})_\alpha^L (\gamma_{kj})_\alpha^L \leq \phi \leq \sum_{k} (\mathcal{R}_{ik})_\alpha^U (\gamma_{kj})_\alpha^U. \]

Therefore,

\[ \min f(\phi) = \frac{\sum_{k=1}^{n} (\mathcal{R}_{ik})_\alpha^L (\gamma_{kj})_\alpha^L}{\varphi + \sum_{k=1}^{n} (\mathcal{R}_{ik})_\alpha^L (\gamma_{kj})_\alpha^L} \]

and

\[ \max f(\phi) = \frac{\sum_{k=1}^{n} (\mathcal{R}_{ik})_\alpha^U (\gamma_{kj})_\alpha^U}{\varphi + \sum_{k=1}^{n} (\mathcal{R}_{ik})_\alpha^U (\gamma_{kj})_\alpha^U} \]

Furthermore,

\[ \sum_{l=1}^{n} \sum_{k=1}^{n} (\mathcal{R}_{ik})_\alpha^L (\gamma_{kl})_\alpha^L \leq \varphi \leq \sum_{l=1}^{n} \sum_{k=1}^{n} (\mathcal{R}_{ik})_\alpha^U (\gamma_{kl})_\alpha^U, \]

such that the modified lower and upper bounds of \( \alpha \)-cuts of \( \mathcal{R}_{ij} \), \( m(\mathcal{R}_{ij})_\alpha^L \) and \( m(\mathcal{R}_{ij})_\alpha^U \), can be formulated as

\[ m(\mathcal{R}_{ij})_\alpha^L = \min f(\phi) = \frac{\sum_{k=1}^{n} (\mathcal{R}_{ik})_\alpha^L (\gamma_{kj})_\alpha^L}{\sum_{l=1}^{n} \sum_{k=1}^{n} (\mathcal{R}_{ik})_\alpha^U (\gamma_{kl})_\alpha^U + \sum_{k=1}^{n} (\mathcal{R}_{ik})_\alpha^L (\gamma_{kj})_\alpha^L}, \]

\[ m(\mathcal{R}_{ij})_\alpha^U = \max f(\phi) = \frac{\sum_{k=1}^{n} (\mathcal{R}_{ik})_\alpha^U (\gamma_{kj})_\alpha^U}{\sum_{l=1}^{n} \sum_{k=1}^{n} (\mathcal{R}_{ik})_\alpha^L (\gamma_{kl})_\alpha^L + \sum_{k=1}^{n} (\mathcal{R}_{ik})_\alpha^U (\gamma_{kj})_\alpha^U}. \]

Comparing equations (6) and (8), obviously the latter has a shorter interval than the former does at each \( \alpha \)-cut. Once the modified lower and upper bounds of \( \alpha \)-cuts of the fuzzy normalized relationship are determined, they are used to find out the fuzzy technical importance ratings of DFLs, which are considered as the priorities of DRs. In the design process of QFD, the fuzzy
technical importance rating set for the \( j \)th DR, \( W_j \), \( j = 1,2,\ldots,n \), is obtained by the sum of multiplying the importance degree of each customer requirement by the corresponding fuzzy normalized relationship as

\[
W_j = \sum_{i=1}^{m} k_i \cdot \tilde{R}_{i,j},
\]

where \( k_i \) is the importance degree of the \( i \)th customer requirement, \( i = 1,2,\ldots,m \). Therefore, the fuzzy technical importance rating is the measure for one design requirement on the overall impact of customer satisfaction. At a specific possibility level \( \alpha \), the lower and upper bounds of the \( \alpha \)-cut of \( W_j \) can be solved as

\[
(W_j)_\alpha = \left( (W_j)_\alpha^L, (W_j)_\alpha^U \right) = \left[ \sum_{i=1}^{m} k_i \cdot m (\tilde{R}_{i,j})_\alpha^L, \sum_{i=1}^{m} k_i \cdot m (\tilde{R}_{i,j})_\alpha^U \right],
\]

which will be used to decide the optimal fulfillment level of each DR.

### 3. FUZZY MODELS

Based on the fuzzy technical importance ratings of DRs, this study formulated a fuzzy linear programming model to determine the fulfillment levels of DRs for finding out the maximum level of customer satisfaction in this section. Although conventional QFD approaches focus on increasing customer satisfaction level, some authors emphasized the need to take into account the costs, technical difficulties, or other organizational constraints in conjunction with the QFD planning efforts \([4,5,12-14,18]\). Wasserman \([14]\) even proposed an expression for maximizing customer satisfaction based on the idea of incremental unit cost. Suppose that a base unit cost can be determined as the need for the product with basic design. In his expressions, the decision variables, \( x_j \), are defined in percentages to denote the level of fulfillment of engineering design requirement \( j \), \( j = 1,2,\ldots,n \). The value of \( x_j \) represents the fulfillment percentage of the objective targets for engineering requirement \( j \), and a corresponding percentage of unit increment cost \( c_j \) which is required to enhance the quality. The total incremental unit cost cannot exceed a cost constraint. Consistent with the expression, a base unit cost is set at \( x_1 = x_2 = \cdots = x_n = 0 \).

As mentioned previously, the relationships between CRs and DRs, as well as correlations among the DRs are fuzzy in nature, such that the technical importance measures are fuzzy. This motivates the development of a fuzzy linear programming model in this study. For involving the imprecision in the cost estimate, the incremental unit cost is also fuzzy in the proposed model. Besides the cost limitation, the impacts of customer satisfaction from different engineering design requirements are prioritized in the model. In addition, the fulfillment level of one DR may be specified in an interval, i.e., \( 0 \leq \varepsilon_j \leq x_j \leq \eta_j \leq 1 \), due to the business competitions and technical difficulty. Involving the above considerations, a fuzzy linear programming model is formulated as

\[
\tilde{Z} = \max \sum_{j=1}^{n} \tilde{W}_j \cdot x_j
\]

s.t. \( \sum_{j=1}^{n} \tilde{C}_j \cdot x_j \leq B \),

\[
\tilde{W_s} \cdot x_s \geq \tilde{W_p} \cdot x_p, \quad s, p \in \{1,2,\ldots,n\},
\]

\[
0 \leq \varepsilon_j \leq x_j \leq \eta_j \leq 1, \quad \forall j,
\]

where \( \tilde{C}_j \) represents the fuzzy cost corresponding to the fulfillment of the \( j \)th engineering design requirement, \( j = 1,2,\ldots,n \), \( B \) is the maximum incremental unit cost limitation, and \( \leq \) signifies the “approximately less than or equal to” relation. Obviously, the objective value \( \tilde{Z} \) is fuzzy,
since each technical importance rating is fuzzy. In order to find the membership function of \( \tilde{Z} \), it suffices to find the lower and upper bounds of the \( \alpha \)-cuts of \( \tilde{Z} \) [19], which can be formulated as

\[
(Z)^{L}_{\alpha} = \min Z,
\]

\[
\text{s.t. } (W_j)^{L}_{\alpha} \leq w_j \leq (W_j)^{U}_{\alpha}, \quad \forall j,
\]

\[
(C_j)^{L}_{\alpha} \leq c_j \leq (C_j)^{U}_{\alpha}, \quad \forall j,
\]

\[
(Z)^{U}_{\alpha} = \max Z,
\]

\[
\text{s.t. } (W_j)^{L}_{\alpha} \leq w_j \leq (W_j)^{U}_{\alpha}, \quad \forall j,
\]

\[
(C_j)^{L}_{\alpha} \leq c_j \leq (C_j)^{U}_{\alpha}, \quad \forall j,
\]

or in full form

\[
(Z)^{L}_{\alpha} = \min \max \sum_{j=1}^{n} w_j \cdot x_j,
\]

\[
\text{s.t. } \sum_{j=1}^{n} c_j \cdot x_j \leq R,
\]

\[
\nu_{sp} \cdot x_s \geq w_p \cdot x_p, \quad s, p \in \{1, 2, \ldots, n\},
\]

\[
0 \leq \varepsilon_j \leq x_j \leq \eta_j \leq 1, \quad \forall j,
\]

\[
(Z)^{U}_{\alpha} = \max \max \sum_{j=1}^{n} w_j \cdot x_j,
\]

\[
\text{s.t. } \sum_{j=1}^{n} c_j \cdot x_j \leq D,
\]

\[
\nu_{sp} \cdot x_s \geq w_p \cdot x_p, \quad s, p \in \{1, 2, \ldots, n\},
\]

\[
0 \leq \varepsilon_j \leq x_j \leq \eta_j \leq 1, \quad \forall j.
\]

For the above formulation, in order to find the smallest objective value at each \( \alpha \)-cut, \( (Z)^{L}_{\alpha} \), we can set the levels of technical importance ratings to their lowest values in the objective function, and the constraints incremental unit costs and the associated technical importance ratings are also specified for tightening the feasible region. On the other hand, the highest objective value, \( (Z)^{U}_{\alpha} \), can be obtained by establishing the formulation with the opposite conditions as \( (Z)^{L}_{\alpha} \) does. Specifically, models (13a),(13b) can be reformulated as

\[
(Z)^{L}_{\alpha} = \max \sum_{j=1}^{n} (W_j)^{L}_{\alpha} \cdot x_j,
\]

\[
\text{s.t. } \sum_{j=1}^{n} (C_j)^{U}_{\alpha} \cdot x_j \leq D,
\]

\[
(W_s)^{L}_{\alpha} \cdot x_s - (W_p)^{U}_{\alpha} \cdot x_p \geq 0, \quad s, p \in \{1, 2, \ldots, n\},
\]

\[
0 \leq \varepsilon_j \leq x_j \leq \eta_j \leq 1, \quad \forall j,
\]
Relationship Symbols

- strong relation
- moderate relation
- weak relation

<table>
<thead>
<tr>
<th>Customer Wants</th>
<th>CR1</th>
<th>CR2</th>
<th>CR3</th>
<th>CR4</th>
</tr>
</thead>
<tbody>
<tr>
<td>CR1</td>
<td>15%</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>CR2</td>
<td>25%</td>
<td></td>
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<tr>
<td>CR3</td>
<td>45%</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>CR4</td>
<td>15%</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Figure 2. The QFD matrix for a writing instrument.

\[
(Z)^U_\alpha = \max \sum_{j=1}^{n} (W_j)^U_\alpha \cdot x_j,
\]

s.t. \[\sum_{j=1}^{n} (C_j)^L_\alpha \cdot x_j \leq B,\]

\[
(W_s)^U_\alpha \cdot x_s - (W_p)^L_\alpha \cdot x_p \geq 0 \quad s, p \in \{1, 2, \ldots, n\},
\]

\[0 \leq \varepsilon_j \leq x_j \leq \eta_j \leq 1, \quad \forall j.
\]

Once the lower and upper bounds of the fuzzy objective function \(\tilde{Z}\) are obtained at several \(\alpha\)-cuts, its membership function can then be established. The maximum range and the most possible value of \(\tilde{Z}\) are useful for the design team in determining the fulfillment levels of design requirements.

4. AN ILLUSTRATIVE EXAMPLE

A simple example of a writing instrument, which appeared in [14], is adopted to exemplify the feasibility of the proposed models in this section. Four customer requirements (CRs) and five design requirements (DRs) are considered in the design. Figure 2 illustrates the HOQ. Here, four CRs include “easy to hold” (CR1), “does not smear” (CR2), “point lasts” (CR3), and “does not roll” (CR4), while the important engineering design requirements contain “length of pencil” (DR1), “time between sharpening” (DR2), “least dust generated” (DR3), “hexagonal” (DR4), and “minimal erasure residue” (DR5).

To derive the fuzzy importance ratings of the five DRs, first, the fuzzy relationships \(\tilde{R}_{ik}\) between CRs and DRs and those \(\tilde{\gamma}_{kj}\) among DRs must be determined. These relationships are judged as any of “none (N)”, “weak (W)”, “moderate (M)”, and “strong (S)”. These linguistic terms are translated into the triangular fuzzy number as \((0,0,0)\), \((0,0.1,0.2)\), \((0.2,0.3,0.4)\), and \((0.8,0.9,1.0)\), respectively. The middle value in the parentheses is the most possible one and has the membership degree equivalent to 1, for example, \(\mu_S(0.9) = 1\). In this study, the system of 0.1-0.3-0.9 is used to represent the most possible value of the weak-medium-strong based on the previous study [14].

Considering the linguistic relationships shown in Figure 2, the fuzzy normalized relationship, \(\hat{R}_{ij}\), can be calculated through the proposed expressions. For obtaining \(\hat{R}_{ij}\), the upper and lower bounds of \(\alpha\)-cuts of \(\tilde{R}_{ik}\) and \(\tilde{\gamma}_{kj}\) should be determined beforehand based on their membership functions. The membership function of a triangular fuzzy number can be defined easily by linear functions. As an illustration, suppose that \(\tilde{R}_{ik}\) is assessed as “strong”, and the
A Fuzzy Model

The membership function of the fuzzy number \( \tilde{S} = (0.8, 0.9, 1.0) \) can be expressed as

\[
\mu_{\tilde{S}}(R_{ik}) = \begin{cases} 
\frac{(R_{ik} - 0.8)}{(0.9 - 0.8)}, & 0.8 \leq R_{ik} \leq 0.9, \\
\frac{(1.0 - R_{ik})}{(1.0 - 0.9)}, & 0.9 \leq R_{ik} \leq 1.0.
\end{cases}
\]

The \( \alpha \)-cuts of the corresponding membership function is

\[
[(R_{ik})^L_\alpha, (R_{ik})^U_\alpha] = [0.8 + 0.1\alpha, 1.0 - 0.1\alpha].
\]

Figure 3. Two illustrated membership functions of \( R_{14} \) and \( R_{44} \).

<table>
<thead>
<tr>
<th>Customer Wants</th>
<th>Degree of Importance (( k_i ))</th>
<th>Engineering Design Requirements</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>DR1</td>
<td>DR2</td>
</tr>
<tr>
<td>CR1 15%</td>
<td>.250</td>
<td></td>
</tr>
<tr>
<td>CR2 25%</td>
<td>.190</td>
<td>.405</td>
</tr>
<tr>
<td>CR3 45%</td>
<td>.023</td>
<td>.185</td>
</tr>
<tr>
<td>CR4 15%</td>
<td>.100</td>
<td></td>
</tr>
</tbody>
</table>

Figure 4. The normalized relationship values for \( \alpha = 1 \).

<table>
<thead>
<tr>
<th>Customer Wants</th>
<th>Degree of Importance (( k_i ))</th>
<th>Engineering Design Requirements</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>DR1</td>
<td>DR2</td>
</tr>
<tr>
<td>CR1 15%</td>
<td>[.208, .292]</td>
<td></td>
</tr>
<tr>
<td>CR2 25%</td>
<td>[.146, .237]</td>
<td>[.353,.461]</td>
</tr>
<tr>
<td>CR3 45%</td>
<td>[.010, .037]</td>
<td>[.141, .234]</td>
</tr>
<tr>
<td>CR4 15%</td>
<td>[.050, .150]</td>
<td></td>
</tr>
</tbody>
</table>

Figure 5. The lower and upper bounds of the normalized relationships for \( \alpha = 0.5 \).

<table>
<thead>
<tr>
<th>Customer Wants</th>
<th>Degree of Importance (( k_i ))</th>
<th>Engineering Design Requirements</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>DR1</td>
<td>DR2</td>
</tr>
</tbody>
</table>

Figure 6. The membership functions of five fuzzy technical importance ratings.
Table 1. The fuzzy cost parameters, and the lower and upper constraints of fulfillment levels.

<table>
<thead>
<tr>
<th>Fulfillment Level</th>
<th>( \tilde{C}_j )</th>
<th>( \alpha )-cut</th>
<th>( \varepsilon_j )</th>
<th>( \eta_j )</th>
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<tbody>
<tr>
<td>( x_1 )</td>
<td>(0.9,1,0,1,1)</td>
<td>[0.9+0.1(\alpha),1.1-0.1(\alpha)]</td>
<td>0.1</td>
<td>1.0</td>
</tr>
<tr>
<td>( x_2 )</td>
<td>(0.5,0.6,0.7)</td>
<td>[0.5+0.1(\alpha),0.7-0.1(\alpha)]</td>
<td>0.1</td>
<td>1.0</td>
</tr>
<tr>
<td>( x_3 )</td>
<td>(0.4,0.5,0.6)</td>
<td>[0.4+0.1(\alpha),0.6-0.1(\alpha)]</td>
<td>0.1</td>
<td>1.0</td>
</tr>
<tr>
<td>( x_4 )</td>
<td>(0.2,0.3,0.4)</td>
<td>[0.2+0.1(\alpha),0.4-0.1(\alpha)]</td>
<td>0.1</td>
<td>1.0</td>
</tr>
<tr>
<td>( x_5 )</td>
<td>(0.4,0.5,0.6)</td>
<td>[0.4+0.1(\alpha),0.6-0.1(\alpha)]</td>
<td>0.1</td>
<td>0.85</td>
</tr>
</tbody>
</table>

Figure 7. The membership functions of \( x_1 \), \( x_2 \), and \( x_5 \) based on the constraints.

Figure 8. The membership functions of fuzzy objective values.

Once the \( \alpha \)-cuts of all relationships are determined, they are placed into the proposed equations for obtaining the upper and lower bounds of \( \alpha \)-cuts of fuzzy normalization relationship. For demonstrations, equations (6) and (8) are applied to find the membership functions (MF14 and MF44) of \( \tilde{R}_{14} \) and \( \tilde{R}_{44} \) at each \( \alpha \) level as shown in Figure 3. In the figure, the solid lines are obtained using equation (6), while the dotted lines are determined through equation (8). Apparently, the dotted lines have narrower spreads for the two membership functions. They will be used for further analysis to acquire more credible results. At one extreme end for the possibility level \( \alpha=1 \), the normalization values of the relationships between CRs and DRs are crisp values and are the most likely ones, shown as in Figure 4. At other possible levels, the normalization values of the relationships definitely appear in the range of the lower and upper bounds of \( \alpha \)-cuts. As illustrated, the normalization values for \( \alpha=0.5 \) are shown in Figure 5.

The importance degrees (\( k_i, \sum k = 100\% \)) of customer requirements from [14], listed in Figure 2, are also adopted to determine the fuzzy technical importance ratings. The fuzzy technical importance rating \( \tilde{W}_j \) is obtained by applying equation (9) to determine the priority of each design requirement. For example, the membership functions of the fuzzy technical importance ratings for the design requirements are shown in Figure 6. The fuzzy technical importance ratings of DR3, DR4, and DR5, i.e., \( \tilde{W}_3 \), \( \tilde{W}_4 \), and \( \tilde{W}_5 \), are ranked higher than the other two. For achieving the maximum customer satisfaction, these ratings are used to formulate the model in determining the optimal fulfillment level of each engineering design requirement. Considering the business competitions, the fulfillment level of each DR is specified to be at least 0.1; in addition,
the level of the fifth DR cannot exceed 0.85 because of the technological difficulty. From the viewpoints of the product, the customer satisfaction contributed from the third DR should not be less than the fourth one, i.e., $\overline{W}_3 \cdot x_3 \geq \overline{W}_4 \cdot x_4$. The increment cost budget is constrained as 2 units. The increment cost of each DR with a complete fulfillment is fuzzy and defined as a triangular fuzzy number. Table 1 lists the minimum and maximum fulfillment level of each DR, the associated fuzzy numbers and $\alpha$-cuts.

Placing the $\alpha$-cuts into the formulation (14), the fulfillment levels of all DRs can be acquired at the $\alpha$ levels. The outcomes of $x_1$ and $x_2$ are fuzzy numbers, as shown in Figure 7, while $x_3$, $x_4$, and $x_5$ are crisp and have 100%, 100%, and 85% fulfillment levels, respectively. At the extreme condition $\alpha = 0$, the fulfillment levels $x_1$ and $x_2$ should be located in the intervals [0.1, 0.62] and [0.54, 1.0], respectively. This implies that the possible fulfillment level of the first DR could not exceed 0.62, while the second DR could not have the fulfillment level less than 0.54. The lower and upper bounds of objective values at each $\alpha$ level can also be determined in the resolution process. Figure 8 illustrates the membership functions of fuzzy objective values. The range of total customer satisfaction level is produced with respect to the $\alpha$ level in determining the fulfillment levels of DRs, and the possible range should belong to [64.9%, 121.3%]. The most possible value is 90.7% under the budget and technique constraints.

5. CONCLUSIONS

Extended models are proposed for aiding the QFD term in determining the fulfillment levels of DRs in the stage of product design by introducing the concept of fuzzy sets, due to the impreciseness of evaluating the relationships between CRs and DRs as well as among the DRs. A modified fuzzy normalization relationship measure is derived to obtain more meaningful representation of fuzzy technical importance ratings of DRs. Considering fuzzy cost parameters, business competition, and technical difficulty of elevating quality, a fuzzy model is formulated to determine the fulfillment level of each DR in order to obtain the maximum total customer satisfaction level at each possibility level. The membership function of fuzzy customer satisfaction is then determined.

Unlike the existing QFD approaches, the proposed approach can allow the QFD team to assess the relationships between CRs and DRs as well as that among the DRs by linguistic terms for dealing with the uncertainty in the stage of product design. The fuzzy numbers are applied in the resolution processes. The possible ranges of the fulfillment levels of DRs and the resulting ranges of total customer satisfaction level can provide a QFD team with more useful information.

REFERENCES