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On the Linear Programming to Bank Portfolio Management

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Abstract: This study considered the application of linear programming to Bank portfolio management. The general linear programming mathematical formulation problem was stated and transform in to standard form, which is apply to Bank portfolio management for decision making in terms of restructuring the economy.

Key words: Linear programming, bank portfolio management, constraints, limitation, analysis of decision variable, analysis of slack and surplus variables

INTRODUCTION

Mathematical programming problems in general are concerned with the use or allocation of scarce resources: labour, materials, machines and capital in the "best" possible manner so that costs are minimized or profits are maximized. In using the term "best" it implies that a set of alternative courses of actions is available for making a decision. Generally, the best decision is found by solving a mathematical problem. The term linear programming merely defines a particular class of mathematical programming problems that meet the following conditions (Taha, 1982; Igbizio, 1982; Bazara and Jarvis, 1977; Dantzig, 1963):

- The decision variables involved in the problem are non-negative.
- The criterion for selecting the "best" values of the decision variables can be described by a linear function these variables, that is, a mathematical function involving only the first power of the variables with no cross products. The criterion function is normally referred to as the objection function.
- The operating rules governing the process, for example, scarcity of resources can be expressed as a set of linear equations or linear inequalities. This set is referred to as the set of constraints.

In an optimization problem, one seeks to either maximize or minimize a specific quantity, called the objective, which depends on a finite number of input variables. These variables may be independent of one another, or they many be related through one or more constraint(s). There are many areas which linear programming has proved useful, few of them are (James, 1974; Hadley, 1962; Anderson *et al*., 1974):

- In the food processing industry, linear programming has been used to determine the optimal shipping plan for the distribution of a particular product from the different manufacturing plants to the various warehouses.
- In the iron and steel industry, the linear programming was used to decide the types of products to be made in their rolling mills to maximize the profit. Metal working industries use linear programming for shop loading and for determining the choice between producing and buying a part. The optimal routine of messages in a communication network and the routing of aircraft and ships can also be decided by using linear programming. In banking system, linear programming can also be applied to portfolio management.

MATHEMATICAL FORMULATION OF LINEAR PROGRAMMING

Linear Programming is an optimization method applicable for the solution of problems in which the objective function and the constraints appear as linear functions of the decision variables. In a more precise form, it can be put to mean a mathematical method for solving a large class of problems in which an attempt is made towards achieving or either maximizing benefit while using limited resources or minimizing cost while using

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certain requirements. Any linear programming problem could be viewed as composed of 3 components namely: Decision variables, Objective functions and the Constraints. The constraint equation in a linear programming problem may be in the form of equalities or inequalities.

The main objectives of every linear programming problem are either to maximize profit or contribution or to minimize cost as specified in the objective function. Thus the general linear programming problem can be stated in the following standard form (Lee *et al*., 1985).

In scalar form minimize:

$$f(x_1, x_2 \ldots x_2) = c_1 x_1 + c_2 x_2 + \ldots + c_n x_n$$

Subject to the constraints

$$\begin{aligned} \alpha_{11} x_1 + \alpha_{12} x_2 \ldots + \alpha_{1n} x_n &= b_1 \\ \alpha_{21} x_1 + \alpha_{22} x_2 \ldots + \alpha_2 x_n &= b_2 \\ \alpha_{m1} x_1 + \alpha_{m2} x_2 \ldots + mnx_n &= b_m \end{aligned} \quad (2.1)$$

This could as well be stated in a more compact form by using the summation sign as:
Minimize

$$F(x_1, x_2, \ldots, x_2 = \sum_{j=1}^{n} c_j x_j \quad (2.0a)$$

Subject to the constraints

$$\sum_{j=1}^{n} a_{ij} x_j = b_i \quad (2.1a)$$

Where:

$$i = 1, 2, 3, \ldots m \quad (2.1b)$$

And

$$j = 1, 2, \ldots, n \quad (2.1c)$$

$$x_1 \geq 0, x_2 \geq 0, \ldots, x_n \geq 0$$

where, c_j, b_j and α_{ij} ($i = 1, 2, \ldots, m$; $j = 1, 2, \ldots, n$) are known constants and x_j are the decision variable with $f(x_j)$ being the objective function.

Problem representation in matrix form:
Minimize

$$C^T X \quad (2.2)$$

Subject to the constraints

$$ax = b \quad (2.2a)$$

and

$$x \geq 0 \quad (2.2b)$$

Where,

$$X = \begin{Bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{Bmatrix}, \quad b = \begin{Bmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{Bmatrix}, \quad c = \begin{Bmatrix} c_1 \\ c_2 \\ \vdots \\ c_n \end{Bmatrix}.$$

$$a = \begin{bmatrix} a_{11} & a_{12} & \ldots\ldots & a_{1n} \\ & & & \vdots \\ & & & \vdots \\ a_{m1} & a_{m2} & \ldots\ldots & a_{mn} \end{bmatrix}.$$

and the superscript T is used to indicate the transpose.

Hence, one could describe the problem representation above as it comprises:

- Objective function which is of minimization types.
- All the constraints are of the equality type.
- All the decision variable are non-negative.

Transformation of linear programming problem into standard form: Any linear programming problem can be put in the standard form by the use of the following transformations:

- The maximization of a function F ($x_1, x_2, \ldots, x_n$) is equivalent to the minimization of the negative of the same function. For example, the objective function

Minimize $f = (c_1 x_1 + c_2 x_2 + \ldots + c_n x_n)$
Is equivalent to;
Maximize $f = -f = -c_1 x_1 + c_2 x_2 - \ldots - c_n x_n$

- In most of the engineering optimization problems, the decision variables represent some physical dimensions and hence the variables x_j's have to be non-negative. However, a variable may be unrestricted in sign in some problems. In such cases, an unrestricted variable (which can take a positive, negative, or zero values) can be written as the difference of 2 non-negative variables. Thus, if x_j is unrestricted in sign, it can be written as $x_j = x'_j - x''_j$ where
- If a constraint appears in the form of a "less than" type of inequality as

$$a_{k1}x_1 + a_{k2}x_2 + \ldots + a_{kn}x_n \leq b_k$$

It can be converted into the equality form by adding non-negative slack variables x_{n+1} as follows:

$$a_{k1}x_1 + a_{k2}x_2 + ... + a_{kn}x_n + x_{n+1} = b_k$$

Similarly, if the constraint is in the form of a "greater than" type of inequality as:

$$a_{k1}x_1 + a_{k2}x_2 + ... + a_{kn}x_n \geq b_k$$

It can be converted into the equality form by subtracting a non-negative surplus variable x_{n+1} as follows:

$$a_{k1}x_1 + a_{k2}x_2 + ... + a_{kn}x_n - x_{n+1} = b_k$$

LINEAR PROGRAMMING AND BANK PORTFOLIO MANAGEMENT (LOANS AND ADVANCES)

Operation research modeling generally is to serve as an aid to better decision making in real life organizations. In this regard, mathematical modeling is only a means to an end; it can not and should not be an end in itself. It can only provide insights for decision makers to understand their problems better and provide guidelines for making better decisions than those based on personal judgment and intuition only. Models can also help evaluate the economic consequences of the decision-makers actions within the existing limitation imposed by scarcity of resources. In this study, the application of linear programming to bank portfolio management would be presented. Portfolio management is a term used to describe the allocation of funds among various investment alternatives. In commercial banking, portfolio management refers to the distribution of available resources among securing investments, loans and other assets (Adekenye, 1986).

Banking in Nigeria, like other parts of the world, are the most regulated of all businesses. They are not free to do what they like with the funds deposited with them. These funds must be managed strictly in accordance with certain statutory requirement laid down by government and monetary authorities. Such regulation may stipulates the percentage of bank deposits that must be held in cash or its equivalents or may be regulation in respect of capital adequacy etc. (CBN, 1990).

Banks also have obligations forwards their customers and shareholders. They must manage their resources in such a way that there are sufficient funds at all times to meet deposit/withdrawals, loan, draw-downs, maturities of borrowings and other cash flows (CBN, 1990).

Banks are accountable to their shareholders who have invested in them with the aim of good returns in the form of future dividends and growth. Consequently, banks need assets which produce income substantially higher than their expenditures in order to record reasonable profits. While, pursuing profit objectives, the assets must be kept at an acceptable level of liquidity so as to meet possible demands from depositors and maintain public confidence at all times. In fact, the need to find an appropriate balance between profitability, risk and liquidity makes the asset selection problem a difficult task. Usually, loans and advances promise the highest rate of return and if banker is completely free to take risks, is not concerned with liquidity and is not subject to any statutory restriction problem are simple. It can be shown, therefore that the objectives of shareholder of banks, depositors, government and in one any authorities are conflicting, consequently, the control problem of bank management, use would over, is the need to reconcile the conflicting bank goals of profitability, liquidity and solvency. Given the increasing competition in the industry in Nigeria, this type of decision making can no longer be based on ad hoc procedures, techniques based on rule of thumb, experience, individual judgment and perhaps, hunches. It is the right of the fore going that this study was proposed.

Model formulation: The study used a short-term (1-year) linear programming planning model to determine the optimal allocation of available funds of any commercial bank in Nigeria to the various investment alternatives subject to government regulatory constraints, funds availability and other constraints. The study then used data from a medium-sized Commercial Bank to apply the mode.

In defining the decision variables, it will be assumed that these are three classes of assets in which the bank can invest loans, securities (non corporate) and liquid assets. The study is only concern about loans, therefore we shall denote loan asset by *i* and these are sown different investment for the loan asset, namely agricultural, real estate and construction, manufacturing, public utilities, exports, transportation and commerce, domestic trade and imports loans. They are denoted by j shown in Table 1.

Definition of decision variables parameters:

X_{ij} = Amount to invest in item j of protfolio i.
R_{ij} = Rate of return of item j of protfolio i.
V_{ij} = Previous year of investment in type j of i.
T = Total deposit.
F = Total amount of funds available for investment.
Y = Total loans and advances in the previous year.

Table 1: Specification of decision varies

Asset (i)	Investment types (j)
Loans	Agricultural
	Real estate and construction
	Manufacturing
	Public utilities
	Exports
	Transportation commerce, domestic trade and imports

Formulation of the objective function and parameters objective function:

$$\text{Max } Z = \Sigma_i \Sigma_j R_{ij} X_{ij} \quad \text{(i)}$$

Equation (i) implies that the objective of the model is the maximization of total return on investment during the planning horizon.

Constraints: Sectoral Allocation of loans constraints

- Agricultural sector constraint

$$x_{11} \Big/ \sum_{j=1}^{7} x_{1j} \geq 0.15 \quad \text{(ii)}$$

Equation (ii) states that the total agricultural loans must be at lest 15% of the loans. This is in conformity with the current monetary policy circular issued by central bank of Nigeria (CBN, 1990).

Linearizing Eq. (ii), we have

$$X_{11} - 0.15\, \Sigma^{7}_{j=1} X_{1j} \geq 0 \quad \text{(iii)}$$

i.e.,

$$X_{11} - 0.15x_{11} - 0.15x_{12} - 0.15x_{13} - 0.15x_{14} - 0.15x_{15} - 0.15x_{16} - 0.15x_{17} \geq 0$$

- Real estate and construction constraints

$$x_{12} \Big/ \sum_{j=1}^{7} x_{1j} \geq 0..06 \quad \text{(v)}$$

This equation means that total loans to real estate and construction must be at least 6% of all loan grated.

Linearizing (v) gives

$$X_{12} - 0.06\, \Sigma^{7}_{j=1} X_{1j} \geq 0 \quad \text{(vi)}$$

i.e.,

$$X_{12} - 0.06x_{11} - 0.06x_{12} - 0.06x_{13} - 0.06x_{14} - 0.06x_{15} - 0.06x_{16} - 0.06x_{17} \geq 0 \quad \text{(vii)}$$

- Manufacturing sector constraint

$$x_{13} \Big/ \sum_{j=1}^{7} x_{1j} \geq 0.44 \quad \text{(viii)}$$

This equation state that total loans to manufacturing must be at least 44% of all loans.

Linearizing Eq. (viii), we have

$$x_{13} - 0.44\sum_{j=1}^{7} x_{1j} \geq 0 \quad \text{(ix)}$$

i.e.,

$$X_{13} - 0.44x_{11} - 0.44x_{12} - 0.44x_{13} - 0.44x_{14} - 0.44x_{15} - 0.44x_{16} - 0.44x_{17} \geq 0 \quad \text{(x)}$$

- Public utilities, Exports, Transportation and Communication Sectors Constraint.

$$\frac{X_{14} + X_{15} + X_{16}}{\sum^{7}_{j=1} X_{1j}} \geq 0.13 \quad \text{(xi)}$$

Equation (xi) means that total loans granted to public utilities, exports, transportation and communication sectors must be at least 13% of all loans.

Leanearizing (xi) gives

$$x_{14} + x_{15} + x_{16} - 0.13\sum_{j=1}^{7} x_{1j} \geq 0 \quad \text{(xii)}$$

Thus

$$X_{14} + x_{15} + x_{16} - 0.13x_{11} - 0.13x_{12} - 0.13x_{13} - 0.13x_{14} - 0.13x_{15} - 0.13x_{16} - 0.13x_{17} \geq 0 \quad \text{(xiii)}$$

- Domestic Trade, imports etc sector constraint

$$x_{17} \Big/ \sum_{j=1}^{7} x_{1j} \leq 0.22 \quad \text{(xiv)}$$

By Eq. xiv, we mean that the loans granted to domestic trade and imports sector can not exceed 22% of all loans, hence linearizing Eq. xiv, we have

i.e.,

$$X_{17} - 0.22x_{11} - 0.22x_{12} - 0.22x_{13} - 0.22x_{14} - 0.22x_{15} - 0.22x_{16} - 0.22x_{17} \leq 0$$

Data collection and estimation: For the model to be solvable, the parameters of the model formulated have to be estimated.

The data used for the study were collected from two sources:

- Central Bank of Nigeria Credit Guidelines.
- The annual report of Equatorial Trust Bank for financial years (1995-1999).

Table 2 shows the data of actual returns on each of the investment for the years (1995-1999) our estimates of the parameter R_{1j} are based on the 5 years average. It is arguable that the more likely forecast of the rate of return can be higher than the average value.

The Table 3 shows the actual table deposit in the bank studied for the years 1995 -1999. It can be observed that there is an increasing trend in the deposit structure. At a result, the use of average value to forecast the deposit for the planning period will be underestimation. Hence, we use the simple annual growth rate concept. This jumped form 16.8% between 1996 and 1997 and then declined to 17.1% between 1997 and 1998 and further to 2.8% between 1998 and 1999. The 1998/99 loan growth rate is due to economic recession. Given government's desire to restructure the economy.

The total amount of funds available for investment was forecast like the total deposit using the simple growth rate concept given the increasing trend observed in the amount of funds available for investment.

Limitation of the model: The model suffers the limitations normally associated with linear programming, among others. Firstly, it assumed that the real-life decision maker has only one objective function to optimize. The nature of banking has multiple conflicting objectives as pointed out. Examples of these are profitability, liquidity, capital, adequately and social responsibility.

Linear Programming is good enough for the analysis of this problem (Table 4). Linear programming modeling assumes decision making under certainty implying that the value of the parameters is known for certain. However, given the performance in the Nigerian economy during the current years of the recession the relative stability in the return on investments of banks. etc. This assumption in the Nigeria context particularly in relation to banking does not seem misplaces. Another major demerit of the model is the assumption of linear relationship between the decision variables and model parameters.

Finally the short planning horizon of the problem is also a limitation. However, the inability to forecast possible government monetary policy changes as embodied in CBN's monetary policy guidelines suggest that planning for shorter planning horizon seems justifies.

Table 2: Rate of returns by investment type and year (Percent)

	1995	1996	1997	1998	1999	5 years average
Agric Loans	5-6	5-6	6-7	6-7	8-9	6
Residential Housing	6	6	7	905	9.5	10
Commercial Housing	7-11	8.5-12	9.5-13	13	13	
Manufacturing	9.5	10.5	11.5	13	13	11.5
Public Utilities	9.5	10.5	11.5	13	13	11.5
Exports	9.5	10.5	11.5	13	13	11.5
Transportation	9.5	10.5	11.5	13	13	11.5
Commerce, Domestic Trade, Imports	11.5	12	13	13	13	12.5

Table 3: Deposit in the Bank for the period of studies 1995-1999

Year	Amount	Annual growth rate (%)
1995	309,715,000	-
1996	361,653,000	16.8
1997	591,109,000	63.5
1998	692,224,000	17.1
1999	711,339,000	2.8

Analysis of decision variables: When the objective function value is decomposed in to relative contributions from the various asset of the banking system, it was fund that the loans and advances portfolio has the highest contribution of about 75% earning from the assets.

A comparison of the optimal solution values Table 5 with the actual performance of the bank for the year study show that the model performed much better in relation to following the prescription of the CBN credit guidelines. While the model recommended that a total of N73.8 million be advance as agricultural loan which is 13% of all loans and advances recommended, the actual performance in the bank for the planning horizon of the study was N49.3 million, 9% of the total of N520.1 million loans and advances. Thus the model performs better, being just 2% less than the minimum level specified by the CBN guidelines while the actual performance is 5.4% below the minimum level.

In the real estate and construction sector both the model and actual performance exceeded the credit minimum allocation. The model's recommendation of N 130.57 million is 16% above the CBN guidelines while the actual investment of N146.6 million is 24% above the CBN requirement. This solution is due to the relatively higher yield in the real estate and construction sector.

In manufacturing sector model solution recommended a total exposure of N211.42 million, approximately 36.3% of total loans and advances of N582. 70%, 7% below the guidelines. The actual performance has a short fall of 15% below the indicating that the model performed better. Allocation to Public Utilities, Exports and Transportation is only 2% below CBN credit

Table 4: Estimation by linear programming

Year	Fund generation from operation (i.e. Earning)	Proceeds from share issues	Proceeds from debenture stock issues	Total deposit	Annual growth rate (%)
1995	7.831	-	7.500	309715	-
1996	14.152	2.300	-	361653	17.5
1997	24.240	2.300	-	391109	61.7
1998	30.548	-	-	692224	17.0
1999	28.403	6.450	-	711339	3.2

Table 5: Model optimal values of loans and advances

Asset type	Optimal value (₦ million)
Agric Loans	73.80
Real Estate Loan	130.51
Manufacturing	211.42
Public Utilities Loans	52.13
Export Loans	0.27
Commerce etc. Loans	102.61
Total	570.80

Table 6: Stock surplus variable in the optimal solution

Constraint to which slack/surplus variables corresponds	Variable value (₦ million)
Commerce, Domestic Trade Constraint (Stock)	6.20
Agricultural Loans Lower bound constraint (Surplus)	39.89
Real Estate/Construction Sectorial Loan	6.20
Manufacturing sector lower bound Constraint (surplus)	66.43
Public Utilities Lower Bound Constraint	51.38

guidelines were as actual performance is over 10% below the guidelines minimum level. The model recommended a value that actually agreed with the maximum for the imports, commerce and domestic trade sector while the actually agreed with the maximum for the imports, commerce and domestic trade sector while the actual performance exceeded this maximum by 1.4%. In general it can be seen from the discussion that the model performed much better than actual particle in attempting to meet the guidelines specified by the CBN.

Analysis of slack and surplus variables: A slack variable in LP represent an unused capacity or an unallocated fund or other resource. A surplus variable on the other hand represents the amount that will need to be provided over and above the existing amount of particular resource, such that the solution values can be attained in practice.

Table 6 shows for example that the slack variable corresponding to the commerce domestic sector etc. constraint is N 6.20 million. This can be inter-protected at the unallocated part of the fund initially set aside for the sector by way of loans and advances given all constraints. This is possible because the sector had been much more favoured in previous years, but the model, now attempting to implement as much as possible the CBN credit guidelines has allocated the exact percentage specified by credit guidelines this time.

A total of N 39.89 million will have to be provided over an above the previous year's value if the total loans for the agricultural sector recommended by the model is to be attained. All other variables in this table can be similarly interpreted.

CONCLUSION AND RECOMMENDATIONS

This study has examined one approach to solving portfolio management problems using linear programming/management science.

However, it has been some limitation, some of which can be rectified in future studies. The following suggestions for further studies are offered:

- The study is based on limited time horizon; a future study can use a multi-period planning horizon spreading over several years to determine optimal portfolio investment and optimal sequence balance sheet positions.
- The effect of bad debts on returns from loans and advances can be taken into consideration in future study has demonstrated that the use of a more analytic and scientific approach to portfolio planning can be better than those practices based mainly on experience and hunches particularly in meeting CBN regulatory restrictions. It is therefore suggested that the banks in Nigeria should endeavour to use linear programming techniques and other scientific approach as aids to decision making.

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