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An optimal sizing method for cogeneration plants

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Abstract

An optimal planning method is proposed of the sizing problem for cogeneration system. Based on mathematical programming theory, equipment capacities are determined optimally so as to minimize the annual total cost in consideration of the plants' annual operational strategy. The sizing problem is formulated as a mixed-integer nonlinear programming problem with the constraints of energy demands, equipment performance characteristics and the energy relationships of the whole system. A numerical example about a gas turbine cogeneration plant in shanghai is given to ascertain the effectiveness of the proposed method.

Keywords: Optimal sizing; Cogeneration; Operational strategy

1. Introduction

As representative of distributing energy supply system, building combined heating and power (BCHP) system have been introduced increasingly for commercial and public purposes in China. In order to utilize their high economical and energy-saving potentials, the system planning, especially the capacities of prime movers is very important. This is because if the capacities of prime movers are underestimated, the effect of introducing cogeneration plants becomes relatively small, and if they are overestimated, the feasibilities decrease. As to the commercial or public buildings, both electricity and thermal energy demands fluctuate seasonally and hourly, so it is very difficult to solve the problem since it is necessary to take account of the plants' annual operational strategies for the variations of load demands.

At present, economical and energy-saving properties are evaluated only for several alternatives to determine plants' sizes, among which the best one is chosen. Additionally, the thermal-following or electric-following strategy is adopted conventionally for the prime movers. These approaches have the disadvantage that the optimal capacities cannot be found and the high economical potentials cannot necessarily be utilized. In fact, the operation of cogeneration system is subjected not only to the variation of load demands, but also to the fuel prices and energy policies as well. Therefore, it is necessary to develop a rational method of determining plants' sizes and operational strategies throughout the year.

However, the variations of system structure and operation mode makes an exhaustive search very difficult, if at all possible, by conventional means [1]. Several computer programs have been developed to aid the designer, which differ from each other with respect to range of applicability and depth of analysis [2–5]. One step further is the application of mathematical optimization procedures for the system planning.

The purpose of this paper is to develop an optimal planning method of determining the sizes of cogeneration plants in consideration of operational strategies by the mathematical planning theory. First, the concept of the optimal planning method is described. Secondly, the optimization problem is formulated concretely for a gas turbine cogeneration system. Lastly, a numerical example is given about a cogeneration plant used for a hospital in Shanghai.

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Nomenclature

a, b, c, d, η	performance characteristic values
C	cold water flow rate, kW
Cc	annual capital cost
Cr	annual energy charges
E	electricity, kW
E_{d}, H_{d}, C_{d}	electricity, heating and cooling load, respectively, kW
E_{elec}	purchased electricity, kW
F	natural gas, m^3/h
F_{gas}	natural gas consumption, m^3
i	interest rate
n	life time of equipment
N	number of equipment
Q	heat flow rate, kW
R	capital recovery factor
T_k	each time period
X_k	binary variable vector
Y_k	continuous variable vector

Greek letters

α	limit ratio of heat disposal
δ	binary variable expressing on/off status of operation

Subscripts

disp	heat disposal
GT	gas turbine generator
GT_RB	utilized part of exhaust gas
HB	gas-fired auxiliary boiler
max	the maximum value
min	the minimum value
RB	waste heat recovery boiler
RE	electric compression refrigerator
RS	steam absorption refrigerator

2. Basic concept of optimal planning method

This paper attempts to solve the optimal sizing problem of cogeneration plants utilizing the method of mathematical programming. The annual total cost is minimized from the viewpoint of long-term economics. This cost is evaluated as the sum of the annual capital cost and the annual running cost. The annual capital cost of each equipment is considered as a function of its capacity. The annual operation cost is evaluated as the sum of each energy charges, mainly the electricity and natural gas consumption. It is obvious that the energy charge is calculated from the system's operational strategy.

As constraints of the planning model, it is mainly to consider the performance characteristics of each equipment and the energy balance relationships of each energy flow to satisfy the load demands. In addition to several representative days throughout the year for hourly energy demands estimation, peak energy demands in summer and winter must also be considered for energy supply during the peak periods. Since the performance characteristics of equipment change with the capacities, they should also be evaluated as functions of the capacities.

Design variables of the model are composed of two parts, namely the equipment capacities of the sizing problem and the variables expressing the operational strategies. Though the capacity of each equipment is selected from a set of discrete values in the practical design, it is regarded as a continuous variable in this study. The operational strategy is expressed by the binary and continuous variables, which represent the on/off status energy flow situation of each equipment, respectively.

The algorithm of optimization procedure is shown in Fig. 1. Initial values are given to each piece of equipments at the beginning and the annual capital cost is evaluated. For a plant with each searching step, the operational strategy is assessed and the annual energy charge is evaluated by the

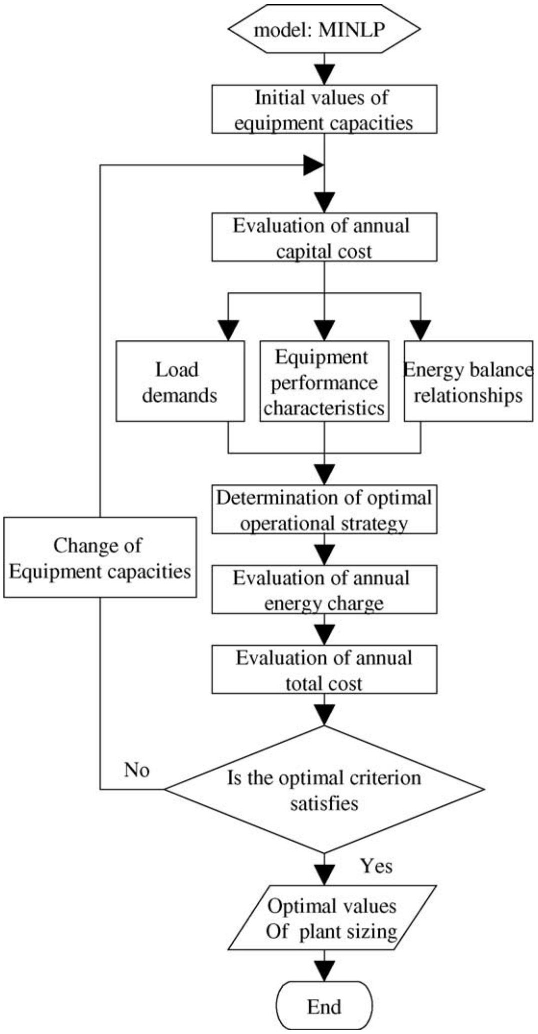

Fig. 1. Algorithm of determining cogeneration plant's size.

comprehensive relationships of load demands, equipment performance characteristics and energy balance of the whole system. Optimal values of equipment capacities are searched so as to minimize the annual total cost. When the optimal criterion is satisfied, the optimal sizing of cogeneration plant in consideration of the annual operational strategy are determined. This problem turned to be a mixed-integer nonlinear programming (MINLP) problem.

3. Formulation of the optimization problem for a gas turbine cogeneration plant

Fig. 2 illustrates the structure of a simple cycle gas turbine cogeneration plant. Equipment symbols are explained in the nomenclature. Although only one unit is illustrated for each kind of equipment, there may be several units. Electricity is supplied to users by the running of gas turbine generator and purchasing electricity from the outside electric power. Purchased electricity is also used to drive electric compression refrigerators. Exhaust heat from the gas turbine is recovered by waste heat recovery boiler. The surplus exhaust heat is disposed of. The shortage of steam is supplemented by gas-fired auxiliary boiler. Cold water for space cooling is supplied by electric-driven and steam absorption chillers. The solid lines, dotted lines, two dots-dash lines, dot-dash lines and broken lines denote the flows of steam, cold water, electricity, exhausted heat and natural gas respectively.

The mathematical model of optimal sizing problem is formulated as follows:

3.1. The objective function

The objective funtion of the optimal sizing problem is the minimization of the annual total cost, which concluding the annual capital cost and the annual energy charge.

$$\mathrm{Min} = \mathrm{Cc} + \mathrm{Cr}. \tag{1}$$

The annual capital cost of equipment Cc is expressed by

$$\mathrm{Cc} = R \times (N_{\mathrm{GT}}C_{\mathrm{GT}} + N_{\mathrm{RB}}C_{\mathrm{RB}} + N_{\mathrm{HB}}C_{\mathrm{HB}} + N_{\mathrm{RS}}C_{\mathrm{RS}} + N_{\mathrm{RE}}C_{\mathrm{RE}}) \tag{2}$$

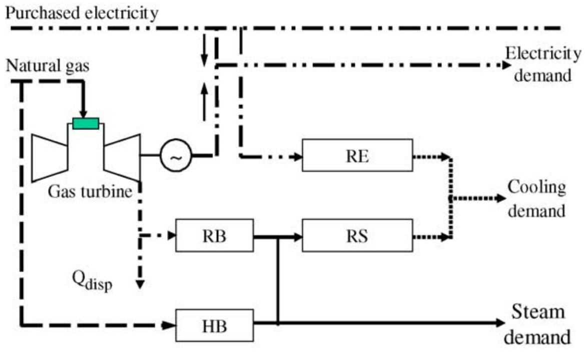

Fig. 2. Plant structure of a gas turbine cogeneration system.

where N and C are respectively the number of pieces of equipment installed and the initial capital cost of each kind of equipment. It is assumed that several pieces of the same capacity are installed for each kind of equipment. The capital recovery factor R is defined by

$$R = \frac{i(1+i)^n}{(1+i)^n - 1} \tag{3}$$

where i is the interest rate and n is the life of the equipment. It is assumed that the values of i and n are equal to all kinds of equipment.

The annual operational hours in a year are discretized by setting D representative days and by dividing each day into H time periods, and T_k is each time period. The annual energy charge Cr is expressed by

$$\mathrm{Cr} = \sum_{D}\sum_{H} C_k(X_k, Y_k)T_k \tag{4}$$

where C_k is the hourly energy charge which is mainly composed of natural gas comsumption and electricity charge. C_k is a function of the binary variable vector X_k and the continuous variable vector Y_k, which express, respectively, on/off status and energy flow rates of each equipment for the corresponding time period T_k.

3.2. The constraints

The constraints of the problem are composed of two parts, namely the performance characteristics of each kind of equipment and the energy flow relationships of the whole system.

Performance characteristics are first formulated for each kind of equipment as follows:

Gas turbine generator

$$f_{\mathrm{GT}} = a_{\mathrm{GT}}\delta_{\mathrm{GT}} + b_{\mathrm{GT}}e_{\mathrm{GT}} \tag{5}$$

$$q_{\mathrm{GT}} = c_{\mathrm{GT}}\delta_{\mathrm{GT}} + d_{\mathrm{GT}}e_{\mathrm{GT}} \tag{6}$$

$$e_{\mathrm{min,GT}}\delta_{\mathrm{GT}} \le e_{\mathrm{GT}} \le 1\delta_{\mathrm{GT}} \quad \delta_{\mathrm{GT}} \in \{0, 1\} \tag{7}$$

$$e_{\mathrm{GT}} = \frac{E_{\mathrm{GT}}}{E_{\mathrm{max,GT}}}, \quad f_{\mathrm{GT}} = \frac{F_{\mathrm{GT}}}{F_{\mathrm{max,GT}}}, \quad q_{\mathrm{GT}} = \frac{Q_{\mathrm{GT}}}{Q_{\mathrm{max,GT}}} \tag{8}$$

$$e_{\mathrm{min,GT}} = \frac{E_{\mathrm{min,GT}}}{E_{\mathrm{max,GT}}} \tag{9}$$

where $E_{\mathrm{max,GT}}, F_{\mathrm{max,GT}}$ and $Q_{\mathrm{max,GT}}$ are respectively the maximum values of electric power, natural gas consumption and heat flow rate of the gas turbine. $E_{\mathrm{min,GT}}$ is the minimum value of electric power, which is considered 50% of $E_{\mathrm{max,GT}}$ in the model.

Waste heat recovery boiler

$$Q_{\mathrm{RB}} = \eta_{\mathrm{RB}}Q_{\mathrm{GT\_RB}}\delta_{\mathrm{RB}} \quad \delta_{\mathrm{RB}} \in \{0, 1\}. \tag{10}$$

Gas-fired auxiliary boiler

$$F_{\mathrm{HB}} = \eta_{\mathrm{HB}} Q_{\mathrm{HB}} \delta_{\mathrm{HB}} \quad \delta_{\mathrm{HB}} \in \{0, 1\}. \tag{11}$$

Steam absorption refrigerator

$$Q_{\mathrm{RS}} = \eta_{\mathrm{RS}} C_{\mathrm{RS}} \delta_{\mathrm{RS}} \quad \delta_{\mathrm{RS}} \in \{0, 1\}. \tag{12}$$

Electric compression refrigerator

$$E_{\mathrm{RE}} = \eta_{\mathrm{RE}} C_{\mathrm{RE}} \delta_{\mathrm{RE}} \quad \delta_{\mathrm{RE}} \in \{0, 1\}. \tag{13}$$

The energy balance of the whole system is formulated for each energy flow as follows:

Electricity

$$E_{\mathrm{elec}} + \sum_{i=1}^{N_{\mathrm{GT}}} E_{\mathrm{GT},i} = \sum_{i=1}^{N_{\mathrm{RE}}} E_{\mathrm{RE},i} + E_d. \tag{14}$$

Exhaust gas

$$\sum_{i=1}^{N_{\mathrm{GT}}} Q_{\mathrm{GT},i} = Q_{\mathrm{disp}} + \sum_{i=1}^{N_{\mathrm{RB}}} Q_{\mathrm{GT\_RB},i}. \tag{15}$$

Steam

$$\sum_{i=1}^{N_{\mathrm{RB}}} Q_{\mathrm{RB},i} + \sum_{i=1}^{N_{\mathrm{HB}}} Q_{\mathrm{HB},i} = \sum_{i=1}^{N_{\mathrm{RS}}} Q_{\mathrm{RS},i} + H_d. \tag{16}$$

Cold water

$$\sum_{i=1}^{N_{\mathrm{RS}}} C_{\mathrm{RS},i} + \sum_{i=1}^{N_{\mathrm{RE}}} C_{\mathrm{RE},i} = C_d. \tag{17}$$

Natural gas

$$F_{\mathrm{gas}} = \sum_{i=1}^{N_{\mathrm{GT}}} F_{\mathrm{GT},i} + \sum_{i=1}^{N_{\mathrm{HB}}} F_{\mathrm{HB},i}. \tag{18}$$

In this optimization algorithm, all the variables will change their values automatically so as to minimize the opjective function in Eq. (1).

4. Numerical example

The effectiveness of the proposed method is illustrated by a numerical example about a gas turbine cogeneration plant for a hospital. Four buildings with different use are to be constructed, and the total floor area is about 23,588 m^2. For simplicity, it is assumed that the plant begins to supply energy to all buildings at the same time. Table 1 shows the annual total and hourly maximum values of electricity, space cooling, and steam demands [6]. Steam is used for space heating and domestic hot water.

Table 1
Annual total and hourly maximum load demands

	Annual total value (MWh)	Hourly maximum value (kW)
Electricity	10869	2086
Space cooling	6281	5360
Steam	11197	4373

Here, one typical day is considered representative day for each month, therefore, the operational strategy is investigated on 12 representative days throughout 1 year. Hourly energy demands are given as input data for each representative day. The load duration curves indicating the annual variations of load demands are illustrated in Fig. 3. In addition to those average load demands, peak energy demands in summer and winter should also be taken into account.

The performance characteristics of each kind of equipments are determined from the actual data using the curve-fitting by the least squares method. Fig. 4 shows the time-of-use rate for purchased electricity. The energy charge for natural gas is 1.9 RMB/m^3 throughout the year. In evaluating the annual capital cost, it is assumed that the interest rate i is 0.1, and the life cycle $n = 15y$.

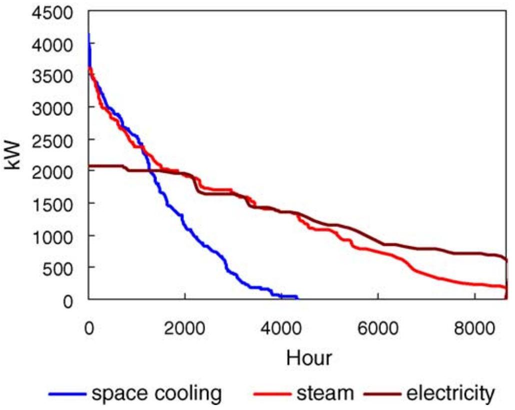

Fig. 3. Load duration curves of energy demands.

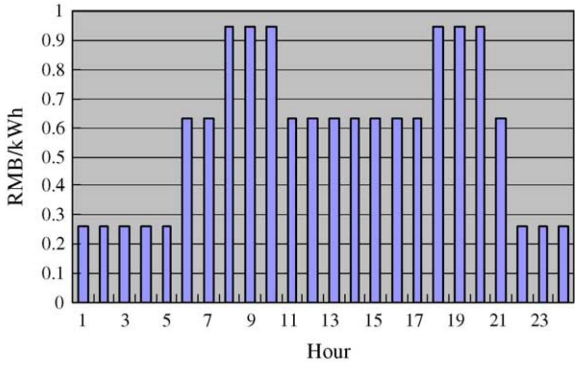

Fig. 4. Time-of-use rate for purchased electricity.

Table 2
Values of equipment capacities

	Case					
	1	2	3	4	5	6
System	Cogeneration					Conventional
Size of gas turbine	Optimal	Optimal	Nonoptimal			–
Size of other equipments	Optimal	Nonoptimal				–
Operational strategy	Optimal	Optimal	Optimal	Electric-following	Heat-following	–
Equipment capacity						
Gas turbine (kW)	1583	1243	1000	1000	1000	–
Waste heat recovery boiler (kg/h)	4866	4214	3747	3747	3747	–
Gas-fired auxiliary boiler (kg/h)	3844	4033	4498	4498	4498	8245
Absorption refrigerator (kW)	2429	3807	3386	3386	3386	–
Compression refrigerator (kW)	2931	1553	1974	1974	1974	5360
Annual capital cost (10^4 RMB)	117.58	110.81	98.94	98.94	98.94	43.69
Annual energy charge (10^4 RMB)	908.50	940.35	977.37	1131.70	1007.95	1088.30
Annual total cost (10^4 RMB)	1026.08	1051.16	1076.31	1230.64	1106.89	1131.99
Reduction rate of annual total cost to that of Case 6 (%)	9.4	7.1	4.9	−8.7	2.2	0

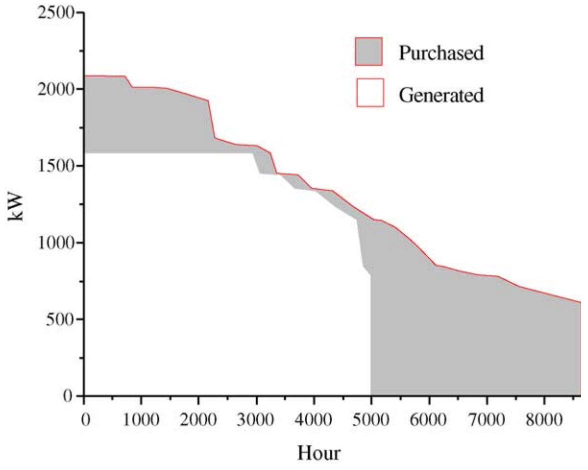

Fig. 5. Load duration curves for electricity supply (Case 1).

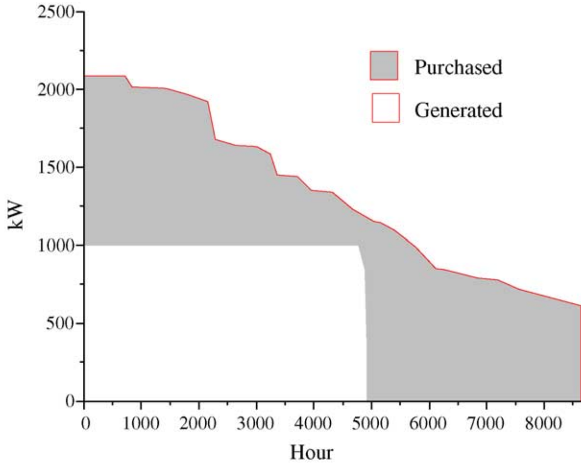

Fig. 6. Load duration curves for electricity supply (Case 3).

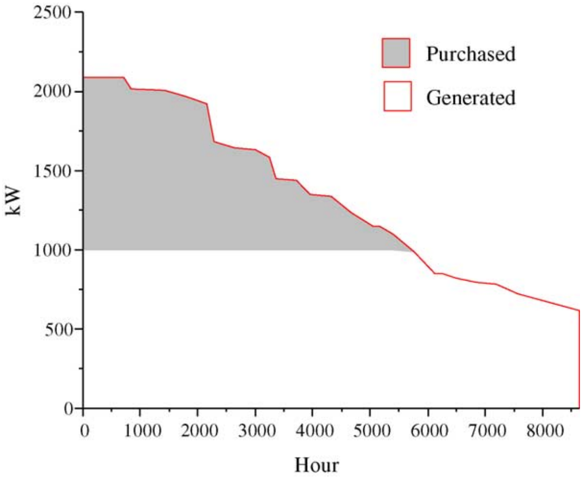

Fig. 7. Load duration curves for electricity supply (Case 4).

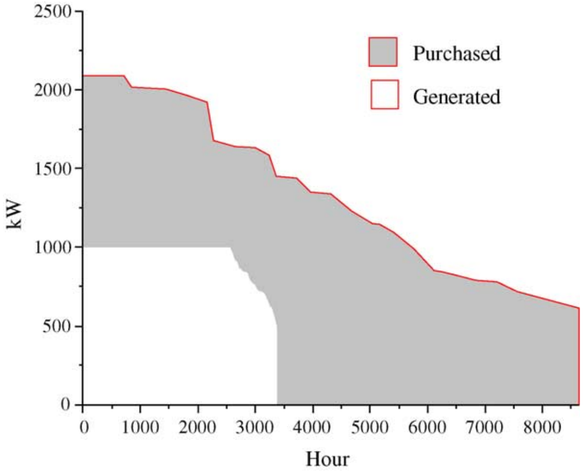

Fig. 8. Load duration curves for electricity supply (Case 5).

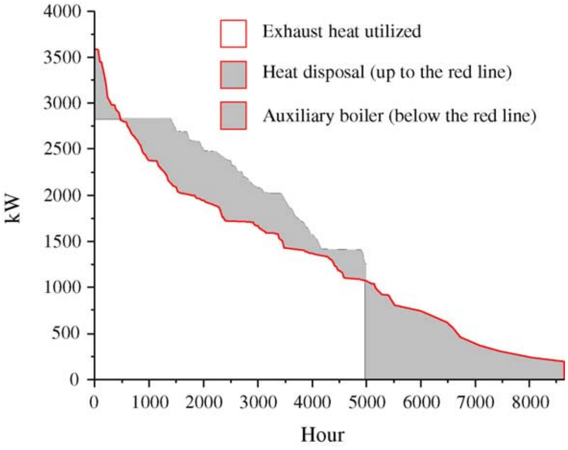

Fig. 9. Load duration curves for heat supply (Case 1).

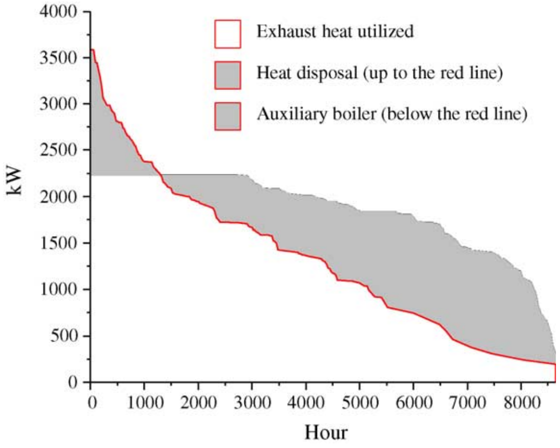

Fig. 11. Load duration curves for heat supply (Case 4).

The energy performance characteristics of cogeneration system are strongly influenced by system structure, equipment sizing and operational strategy. The following six cases are to be studied:

Case 1: cogeneration plant, all kinds of equipments' size determined optimally by considering optimal operational strategy.
Case 2: cogeneration plant, only the size of gas turbine determined optimally by considering optimal operational strategy.
Case 3: cogeneration plant with its size given a priori by considering optimal operational strategy.
Case 4: cogeneration plant with its size given a priori by using the conventional electric-following strategy.
Case 5: cogeneration plant with its size given a priori by using the conventional heat-following strategy.
Case 6: conventional plant composed of gas-fired auxiliary boiler and electric compression refrigerator.

It is assumed that one unit is installed for all kinds of equipment in all cases.

Table 2 shows the values of equipment capacities in all cases and the annual total cost and its items for each case. Load duration curves for electricity and heat supply are shown in Figs. 5–12.

By optimizing both the plant's size and operational strategy, Case 1 ranks the first of all cases with the reduction rate of annual total cost 9.4% to that of the conventional Case 6. It is owing to the optimal operational strategy that leads to the reduction of the energy charge in spite of its highest capital cost. In Case 2, the capacities of equipments other than the gas turbine generator are determined by the conventional method, which lead to the reduction rate of 7.1% by considering the optimal operational strategy.

Case 4 is the worst case, the annual total cost of which has the surplus of 8.7% to the conventional Case 6. This is because by adopting the electric-following operational strategy, a large amount of exhaust heat is disposed of which leads to the decreasing of economic merits. Fig. 11 is the load duration curve for heat supply of Case 4. Therefore, cogeneretion plant is not always better than conventional system at any time. Unreasonable sizing and poor operational strategy will lead to the contrary result.

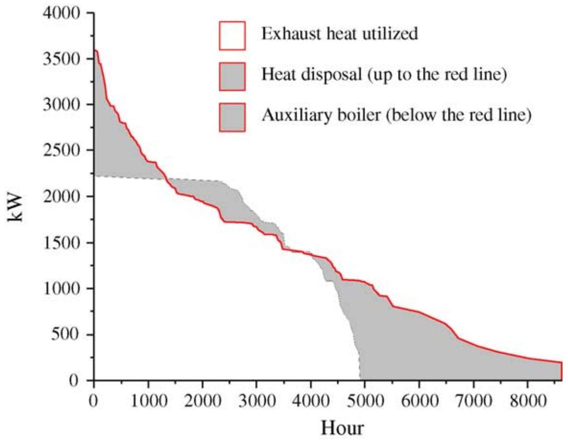

Fig. 10. Load duration curves for heat supply (Case 3).

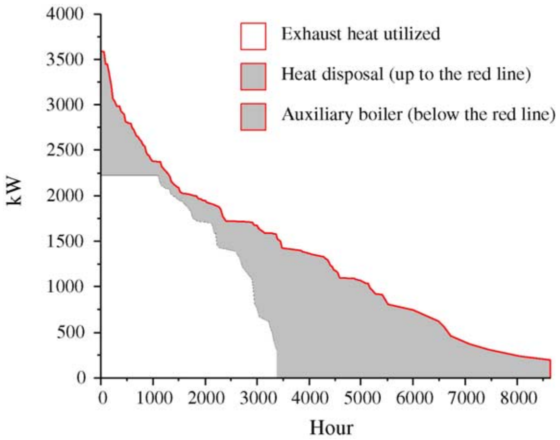

Fig. 12. Load duration curves for heat supply (Case 5).

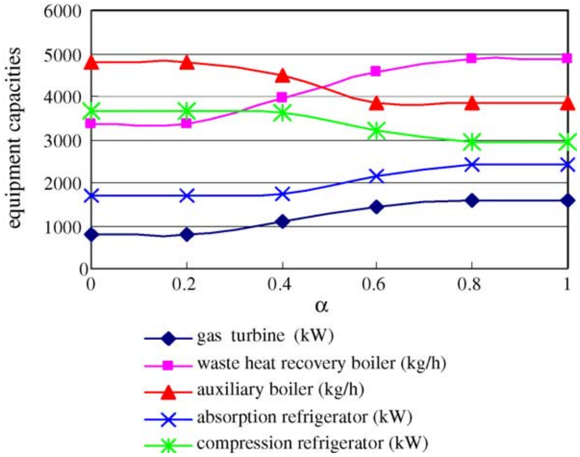

Fig. 13. Relationship between heat disposal limit and optimal values of equipment capacities.

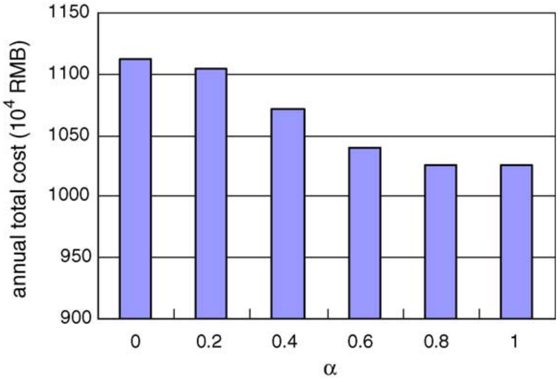

Fig. 14. Relationship between heat disposal limit and annual total cost.

Compared to Case 4, Case 5 adopts the opposite heat-following operational strategy in which the prime mover has been set to trace the heat load demand of the building. Although no exhaust heat has been disposed of, the overall economic merits has only a reduction rate of 2.2% as compared to Case 6. Here a conclusion can be drawn that under some circumstances the disposing of exhaust heat has the advantages from the economic viewpoint.

Although the result shown above is determined optimally from the viewpoint of economics, it is not satisfactory from the energy saving viewpoint because exhaust heat had to be disposed of sometimes. If constraint of heat disposal is added to the problem, the optimization calculation must be carried out again. The constraint that limits the heat diposal is:

$$Q_{\text{disp}} \leq \alpha Q_{\text{GT}}. \tag{19}$$

In Eq. (19), α is the limit ratio of heat disposal to exhaust heat from the gas turbine. Fig. 13 shows the effect of α on the optimal sizing of equipment capacities. The capacities of gas turbine generators increases with the limit ratio α. At the point of $\alpha = 1$ that is no limit of heat disposal, the capacity of gas turbine is nearly twice of that of $\alpha = 0$. As seen from Fig. 14, the annual total cost decreases when the limit ratio α increases. The result shows that the optimal solution only from the economic viewpoint conflicts with the energy saving properties to some extent.

5. Conclusions

An optimal planning method for cogeneration plants has been proposed on the basis of mathematical programming theory. Optimal values of equipment capacities have been determined in considering operational strategy. Through a numerical example about a simple cycle gas turbine cogeneration plant, the effectiveness of the proposed method has been proved. It has also proved that optimal sizing and rational operation are very important to achieve the maximum economic merits of cogeneration system.

Although only a simple cycle gas turbine cogeneration plant is introduced in this paper, the proposed method can be applied not only to other types of gas turbine plants but to plants with different prime movers as well. It turned out to be flexible and effective in the fundamental design of cogeneration plants.

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