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Analysis of the benefit generated by using fuzzy numbers in a TOPSIS model developed for machine tool selection problems

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ABSTRACT

Selection of the appropriate machine tools for a manufacturing company is a very important but at the same time a complex and difficult problem because of the availability of wide-ranging alternatives and similarities among machine tools. In the literature, various machine tool selection procedures are developed. The developed procedures mainly use Multi Criteria Decision Making (MCDM) methods. In the literature, fuzzy MCDM models, in which fuzzy numbers are used instead of crisp values, are proposed to deal with the vagueness and imprecision inherent in the machine tool selection problem. Although, the available studies in the literature developed various fuzzy models, they do not propose any approaches to measure the benefit generated by incorporating fuzziness in their selection models. This paper aims to fill this gap by trying to quantify the level of benefit provided by employing the fuzzy numbers in the MCDM models. Technique for Order Preference by Similarity to Ideal Solution (TOPSIS) is used as the MCDM approach to rank the machine tools in this paper. In the paper, by increasing the fuzziness level steadily in the fuzzy numbers, the obtained machine tool rankings are compared with the ranking obtained with the crisp values. The statistical significance of the differences between the ranks is calculated using Spearman's rank-correlation coefficient. It can be observed from the results that as the vagueness and imprecison increases, fuzzy numbers instead of crisp numbers should be used. On the other hand, in sitiuations where there is a low level of fuzziness or the average value of the fuzzy number can be guessed, using crisp numbers will be more than adequate.

1. Introduction

For manufacturing companies, selection of the appropriate machine tools is very important in achieving high competitiveness in their markets. Machine tools are commonly selected by evaluating qualitative and quantitative criteria such as table size, axis movement, power, spindle speed, axis speed, tool number, machine size and machine cost, work piece material, work piece size, work piece complexity, material removal rate, finish tolerances, process type, etc. (Kalpakjian and Schmid, 2001; Arslan et al., 2004; Sun, 2002). In the literature, there are various papers that proposed models to solve machine tool selection problems. For example, Arslan et al. (2004), Lin and Yang (1996), Oeltjenbruns et al.

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(1995) and Yurdakul (2004) proposed Analytic Hierarchy Process (AHP) for machine tool selection problem. Vasilash (1997) developed a computer program called "machine tool selector" which obtains a feasible set of machine tools by searching the data base and eliminating unsuitable ones. Georgakellos (2005) uses a scoring model that incorporates technical and commercial characteristics of machines in their approach.

However, the studies mentioned above use crisp (non-fuzzy) Multi Criteria Decision Making (MCDM) approaches and do not take into account the uncertainties and imprecision that may be associated with the decision-maker's judgments. Fuzzy MCDM approaches are proposed for selection problems where vagueness and imprecision are involved in the literature. For example, Chu and Lin (2003) proposed a fuzzy Technique for Order Preference by Similarity to Ideal Solution (TOPSIS) model for robot selection, and Wang et al. (2000) developed a fuzzy MCDM model to assist the decision-maker to deal with the machine selection problem for a flexible manufacturing cell. In other studies, Pegero and Rangone (1998) and Jiang and Hsu (2003) used fuzzy AHP for selection of advanced manufacturing technologies, and Devedzic and Pap (1999) presented a study that selects the most rigid machine tool among the alternatives by using a fuzzy linguistic approach.

Although detailed descriptions are provided in the literature, various issues of the fuzzy MCDM approaches are not explored yet. Such a key issue is the justification of the usage of fuzzy versions of the MCDM approaches. A study that compares usage of the fuzzy and crisp numbers or one that provides recommendations about when one is preferred over the other is not available in the literature. This paper tries to fill this gap by determining a level of fuzziness (a threshold value) that warns the users to start using fuzzy MCDM approach instead of its crisp version. It is expected that above the calculated threshold value, the level of uncertainties and imprecision is high enough to require the usage of fuzzy MCDM approaches. On the other hand, a fuzziness level below the threshold value indicates that the benefit of the usage of fuzzy numbers is minimal. The benefit of using fuzzy numbers instead of crisp ones can be measured with the statistical significance of the difference between the rankings obtained using fuzzy numbers and crisp values. When the statistical significance of the difference is above a pre-defined value, it is recommended to use the fuzzy numbers instead of crisp ones. In this study, the measure of fuzziness level is first defined and then various cases are developed by varying (steadily increasing) the level of fuzziness and compared with the ranking obtained using crisp numbers.

Although there are many different fuzzy MCDM approaches to calculate the rating scores and rankings of the machine tools, the Technique for Order Preference by Similarity to Ideal Solution, developed by Hwang and Yoon in 1981, will be used as the ranking method in this paper. The advantage of this method is its simplicity and ability to yield an indisputable preference order (Feng and Wang, 2000; Sen and Yang, 1998). Steps and applicational details of the fuzzy TOPSIS approach are presented in Appendix A (Chu and Lin, 2003; Sen and Yang, 1998; Chen, 2000; Byun and Lee, 2004). The rankings are obtained with the application of the TOPSIS approach for various cases. For each case, the

Table 1 – Machine tools' characteristic values at the selection criteria

Mark	Model	Table area (mm^2)	Spindle speed (rpm)	Power (kW)	Tool number	Tool change time (s)	Max. tool diameter (mm)	Positioning accuracy (10^{-3} mm)
Mazak	FH6000	250,000	10,000	37	40	5	135	2
Okuma	MA500HB	250,000	6,000	30	40	4	140	4
Matsuura	HMAX500	250,000	15,000	22	60	3	80	2
Mazak	PFH5800	250,000	15,000	22	40	3	95	2
Moriseiki	NVD4000DCG	315,000	12,000	19	20	3	80	1
Dahlil	MCH500	250,000	3,500	19	60	8	110	1
Hyundai	SPTV550D	266,500	10,000	16	24	4	90	5
Excel	PM610T32	292,741	8,000	16	33	5	95	5
Challenger	MCV2416	273,600	15,000	8	16	6	90	5
Leadwel	MH500	250,000	4,000	15	40	13	125	1
Eagle	VMC600	320,000	10,000	11	22	5	89	5
Excel	PMCT18	303,386	8,000	10	19	5	95	5
Awea	AV610	315,000	8,000	8	20	6	100	5
Challenger	MCV2412	273,600	8,000	8	16	6	80	5
Taksan	TMC500/1	300,000	6,000	7	16	6	80	5
Dahlil	MCV510	287,000	8,000	8	16	8	90	1

statistical significance of the difference between the ranking obtained for the fuzzy criteria weights and the one obtained for the crisp weight values is determined using Spearman's rank-correlation test. Spearman's rank-correlation test, which is a special form of correlation test, is used when 'the actual values of paired data are substituted with the ranks which the values occupy in the respective samples' (Miller et al., 1990). In this study, Spearman's test evaluates the similarity of the outcomes (rankings of the machine tools for various cases) of the TOPSIS approach. In its application in the paper, to test the null hypothesis (H_0: There is no similarity between the two rankings), a test statistic, Z, is calculated using Eqs. (1) and (2) and compared with a pre-determined level of significance α value. For example, if 3.5, which corresponds to the critical Z-value at the level of significance of $\alpha = 0.0002$, is selected and the test statistic computed by Eq. (2) exceeds 3.5, the null hypothesis is rejected and it is to be concluded that 'H_1: The two rankings are similar' is true. Z-value itself can also be used as a measure of similarity of rankings. A higher Z-value shows a higher similarity between any two rankings.

$$r_s = 1 - \left[\frac{6\sum_{j=1}^{K} {d_j}^2}{K(K^2-1)}\right] \tag{1}$$

$$Z = r_s\sqrt{K-1} \tag{2}$$

In Eqs. (1) and (2), d_j is the ranking difference of alternatives j and K is the number of alternatives to be compared. r_s represents the Spearman's rank-correlation coefficient in Eqs. (1) and (2).

2. Application of the fuzzy TOPSIS and Spearman's rank-correlation approaches for different levels of fuzziness

An example is developed to explain and illustrate the analysis of fuzziness. Sixteen machines and seven criteria are selected for the example, and the machines' performance values at the selected criteria are provided in Table 1. TOPSIS approach (Appendix A) requires the weights of criteria and machine tool performance values at the criteria as inputs. It should be noted that, in the application of the TOPSIS approach, since the machine tools' performance values at criteria are crisp values (Table 1), only criteria weights are required to be fuzzy numbers.

2.1. *Analysis of the fuzziness in terms of spread only while keeping the center of the fuzzy numbers constant*

In the first part of Section 2, six cases (Cases B–G) are developed by varying the lower and upper values of fuzzy numbers while keeping their centers (mean or average) constant at their crisp values given in Case A (Table 2). The level of fuzziness is determined with the 'spread' in the developed cases. The spread is defined in this paper as the number of units that the lower and upper values apart in a trapezoidal or triangular fuzzy number and takes the values 0, 1, 2, ..., 6 in this study. It is assumed that as the level of imprecision and uncer-

Table 2 – Criteria weights for seven different Cases (A–G)

Criteria weights	Crisp number, Case A spread value: 0	Triangular fuzzy number, Case B spread value: 1	Triangular fuzzy number, Case C spread value: 2	Trapezoidal fuzzy number, Case D spread value: 3	Trapezoidal fuzzy number, Case E spread value: 4	Trapezoidal fuzzy number, Case F spread value: 5	Trapezoidal fuzzy number, Case G spread value: 6
Table area (mm^2)	9	8,9,9	8,9,10	7,8,9,10	6,8,9,10	5,7,9,10	4,7,9,10
Maximum spindle Speed (rpm)	8	7,8,8	7,8,9	6,7,8,9	5,7,8,9	5,6,9,10	3,5,8,9
Power (kW)	2	1,2,2	1,2,3	1,2,3,4	1,2,3,5	1,2,3,6	1,2,4,7
Tool number	3	2,3,3	2,3,4	2,3,4,5	1,3,4,5	1,2,3,6	1,2,5,7
Tool change time (s)	4	3,4,4	3,4,5	3,4,5,6	2,4,5,6	2,3,4,7	2,4,7,8
Maximum tool diameter (mm)	5	4,5,5	4,5,6	4,5,6,7	3,5,6,7	4,5,7,9	3,5,8,9
Positioning accuracy (10^{-3} mm)	6	5,6,6	5,6,7	5,6,7,8	4,6,7,8	3,4,7,8	3,4,7,9

Table 3 – Scores and ranks of machine tools for Cases A–G

Machine tool	Model	Case A		Case B		Case C		Case D		Case E		Case F		Case G	
		S.	R.	S.	R.	S.	R.	S.	R.	S.	R.	S.	R.	S.	R.
Mazak	FH6000	0.650	4	0.646	4	0.650	4	0.670	3	0.674	3	0.677	3	0.705	3
Okuma	MA500HB	0.458	9	0.451	9	0.458	9	0.490	6	0.491	6	0.498	6	0.561	5
Matsuura	HMAX500	0.749	1	0.750	1	0.749	1	0.740	1	0.734	1	0.713	2	0.716	1
Mazak	PFH5800	0.746	2	0.749	2	0.746	2	0.734	2	0.729	2	0.718	1	0.711	2
Moriseiki	NVD4000DCG	0.675	3	0.680	3	0.675	3	0.660	4	0.658	4	0.636	4	0.633	4
Dahlil	MCH500	0.444	10	0.439	10	0.444	10	0.477	8	0.473	7	0.445	9	0.496	6
Hyundai	SPTV550D	0.471	7	0.471	7	0.471	7	0.465	9	0.461	9	0.465	7	0.484	7
Excel	PM610T32	0.417	11	0.415	11	0.417	11	0.422	11	0.417	11	0.417	11	0.453	9
Challenger	MCV2416	0.543	5	0.549	5	0.543	5	0.508	5	0.504	5	0.516	5	0.475	8
Leadwel	MH500	0.377	14	0.376	14	0.377	14	0.393	13	0.392	12	0.373	13	0.375	14
Eagle	VMC600	0.460	8	0.462	8	0.460	8	0.446	10	0.441	10	0.443	10	0.450	10
Excel	PMCT18	0.398	12	0.398	12	0.398	12	0.396	12	0.390	13	0.388	12	0.416	12
Awea	AV610	0.383	13	0.384	13	0.383	13	0.378	14	0.372	14	0.371	14	0.389	13
Challenger	MCV2412	0.357	15	0.358	15	0.357	15	0.353	15	0.348	15	0.344	15	0.366	15
Taksan	TMC500/1	0.313	16	0.312	16	0.313	16	0.319	16	0.313	16	0.304	16	0.343	16
Dahlil	MCV510	0.481	6	0.485	6	0.481	6	0.478	7	0.473	8	0.446	8	0.440	11

S., Score and R., rank.

tainty increases, this increase will translate into an increase in the spread (difference) between the lower and upper values of the fuzzy number. For each case, the criteria weights are readjusted by equaling the number of units upper and lower values are apart with the 'spread value' given for that case.

For each case, the ranking scores and rankings are obtained for machine tools using the criteria weights (Table 2) and machine tools' performance values at the selection criteria (Table 1) and provided in Table 3. The obtained rankings for Cases B–G are compared with the ranking obtained for Case A (Table 4). The comparison is performed by taking the difference of the ranks of the machine tools (the columns under the headings 'A–B', 'A–C', . . ., 'A–G' in Table 4) and then calculating Spearman's correlation coefficients (Z-values) for each difference (last row in Table 4).

The calculated Z-values are further illustrated in Fig. 1. It can be observed from the figure that as the level of fuzziness increases, the similarity of the rankings decreases. However, even the lowest Z-value in Fig. 1, 3.44, corresponds to the level of significance $\alpha = 0.0003$ and indicates that using fuzzy numbers instead of crisp values does not provide any meaningful differences in the rankings of Cases B–G compared to Case A. This observation is supported by checking the differences of the ranks of the machine tools provided in Table 4. Cases B and C provided the exact ranking with Case A. For Cases D–F, the highest difference in rankings is 3 for OKUMA MA500HB. The rank difference increases only to 5 when the spread is increased to 6 (Case G). To conclude the benefit resulted from changing the spread without changing the center of the fuzzy number is statistically insignificant; in such cases the user can use crisp values as the criteria weights confidently.

Table 4 – The differences and correlation values of the Cases B–G compared with Case A

		A–B	A–C	A–D	A–E	A–F	A–G
Mazak	FH6000	0	0	1	1	1	1
Okuma	MA500HB	0	0	3	3	3	4
Matsuura	HMAX500	0	0	0	0	−1	0
Mazak	PFH5800	0	0	0	0	1	0
Moriseiki	NVD4000DCG	0	0	−1	−1	−1	−1
Dahlil	MCH500	0	0	2	3	1	4
Hyundai	SPTV550D	0	0	−2	−2	0	0
Excel	PM610T32	0	0	0	0	0	2
Challenger	MCV2416	0	0	0	0	0	−3
Leadwel	MH500	0	0	1	2	1	0
Eagle	VMC600	0	0	−2	−2	−2	−2
Excel	PMCT18	0	0	0	−1	0	0
Awea	AV610	0	0	−1	−1	−1	0
Challenger	MCV2412	0	0	0	0	0	0
Taksan	TMC500/1	0	0	0	0	0	0
Dahlil	MCV510	0	0	−1	−2	−2	−5
$(d^k)^2$		0	0	26	38	24	76
r_s		1.000	1.000	0.962	0.944	0.965	0.888
Z		3.873	3.873	3.725	3.657	3.736	3.440

Table 5 – The fuzzy criteria weights for new Cases (H–Y)

	Case A spread value: 0	Case H spread value: 1	Case I spread value: 2	Case J spread value: 3	Case K spread value: 4	Case L spread value: 5	Case M spread value: 6
Shifted to right							
Table area (mm^2)	9	8,10,10	9,10,10	7,8,10,10	6,7,10,10	5,7,10,10	4,7,10,10
Maximum spindle speed (rpm)	8	7,9,9	7,9,10	6,7,9,10	5,7,9,10	5,6,10,10	3,5,9,10
Power (kW)	2	1,3,3	1,3,4	1,2,4,5	1,2,4,6	1,2,4,7	1,2,5,8
Tool number	3	2,4,4	2,4,5	2,3,5,6	1,3,5,6	1,2,4,7	1,2,6,9
Tool change time (s)	4	3,5,5	3,5,6	3,4,6,7	2,4,6,7	2,3,5,8	2,4,8,9
Maximum tool diameter (mm)	5	4,6,6	4,6,7	4,5,7,8	3,5,7,8	4,5,8,10	3,5,9,10
Positioning accuracy (10^{-3} mm)	6	5,7,7	5,7,8	5,6,8,9	4,6,8,9	3,4,8,9	3,4,8,10
	Case A spread value: 0	Case N spread value: 1	Case O spread value: 2	Case P spread value: 3	Case Q spread value: 4	Case R spread value: 5	Case S spread value: 6
Shifted to left							
Table area (mm^2)	9	6,7,9	6,7,10	5,6,9,10	4,6,7,10	3,5,7,10	2,5,7,10
Maximum spindle speed (rpm)	8	5,6,8	5,6,9	4,5,8,9	3,5,8,9	3,4,7,10	1,3,6,9
Power (kW)	2	1,2,2	1,2,3	1,2,3,4	1,2,3,5	1,2,3,6	1,2,4,7
Tool number	3	2,3,3	2,3,4	2,3,4,5	1,3,4,5	1,2,3,6	1,2,5,7
Tool change time (s)	4	3,4,4	3,4,5	3,4,5,6	2,4,5,6	2,3,4,7	2,4,7,8
Maximum tool diameter (mm)	5	2,3,5	2,3,6	2,3,6,7	1,3,4,7	2,3,5,9	1,3,6,9
Positioning accuracy (10^{-3} mm)	6	3,4,6	3,3,7	3,4,7,8	2,3,4,8	2,2,5,8	1,2,5,9
	Case A spread value: 0	Case T spread value: 1	Case U spread value: 2	Case V spread value: 3	Case W spread value: 4	Case X spread value: 5	Case Y spread value: 6
Shifted to both directions							
Table area (mm^2)	9	6,7,9	6,7,10	5,6,9,10	4,6,9,10	3,5,7,10	2,5,7,10
Maximum spindle speed (rpm)	8	5,6,8	5,6,9	4,5,8,9	3,5,8,9	3,4,7,10	1,3,6,9
Power (kW)	2	1,3,3	1,3,4	1,2,4,5	1,2,4,6	1,2,4,7	1,2,5,8
Tool number	3	2,4,4	2,4,5	2,3,5,6	1,3,5,6	1,2,4,7	1,2,6,9
Tool change time (s)	4	3,5,5	3,5,6	3,4,6,7	2,4,6,7	2,3,5,8	2,4,8,9
Maximum tool diameter (mm)	5	4,6,6	4,6,7	4,5,7,8	3,5,7,8	4,5,8,10	3,5,9,10
Positioning accuracy (10^{-3} mm)	6	3,4,6	3,4,7	3,4,7,8	2,4,5,8	2,2,5,8	1,2,5,9

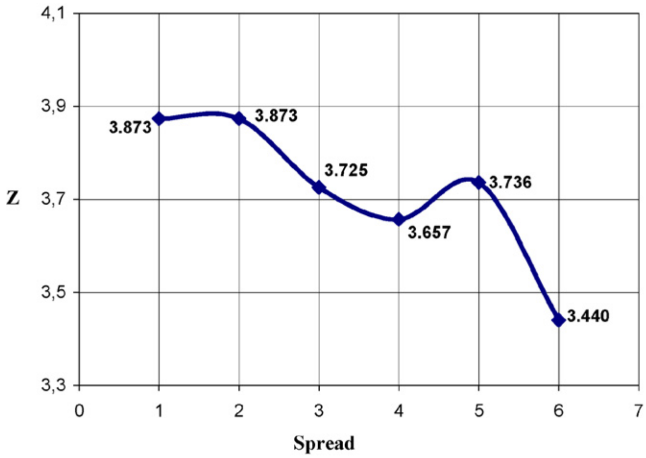

Fig. 1 – Graphic illustration of Spearman rank-correlation test results for Cases A–G.

2.2. *Analysis of the fuzziness in terms of spread and center (average value) together in the fuzzy numbers*

In the cases developed so far, it is assumed that the average values are known with certainty but the users are not sure about the spreads of the numbers. However, in different situations, the average values may also be unknown along with the spread of the fuzzy number which implies increased uncertainties and imprecision. To incorporate the shift in the average values of the fuzzy numbers, new cases are developed (Table 5). In the new cases, the fuzzy numbers are shifted to either left (Cases H–M) or right (Cases N–S) or both directions (Cases T–Y) while the spread is again increased steadily. New cases are again compared with Case A whose crisp criteria weights are not changed from the values provided in Table 2. The calculated Z-values along with the Z-values for the Cases B–G are provided in Fig. 2.

The comparisons of the Z-values obtained for 'shifted to left', 'shifted to right' and 'shifted to both directions' with the 'no-shift' criteria weights (Cases A–G) show that as the fuzziness, which is represented with the shift of the average and the spread together, increases, the similarity of the ranking with the one obtained with the crisp numbers decreases. However, the similarity of the rankings in left-shift and right-shift cases only worsens, when the spread value is equal to 6; i.e. there is a sharp decrease in Z-value when the spread value is 6. For one sided shift cases, the lowest Z-value is above 3.246 which leads to the conclusion that even when the numbers are shifted, the differences in the rankings are not statistically significant. On the other hand, shifting some criteria weights left and shifting others in a fuzzy number worsened the similarity between rankings, but the lowest Z-value is still above 3, which corresponds to the level of significance $\alpha = 0.0013$.

It can be concluded that the statistical significance of the benefit, which is defined in terms of the difference between the ranking obtained with the fuzzy numbers and the one obtained with the crisp values, is minimal. Especially the user can use crisp criteria weights confidently at low fuzziness levels. On the other hand, the magnitude of benefit of using fuzzy numbers increases when the decision maker is not sure about the spread and the mean of the fuzzy number at the same time. In such a case, although the statistical significance values of the differences in machine tool rankings are still very small, using fuzzy numbers are recommended to quantify the criteria weights. Even small changes in the rankings may lead to the elimination of the best machine and selection of a less

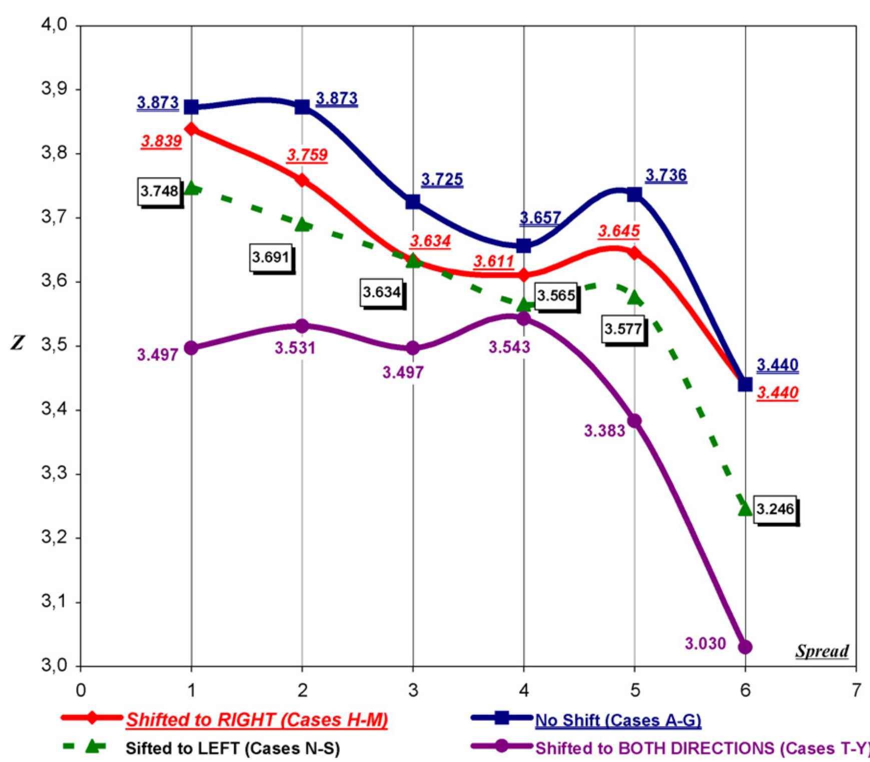

Fig. 2 – Z-values for Cases A–Y.

qualified machine tool.

3. Conclusion

This paper aimed to measure the benefit of using fuzzy numbers instead of crisp ones in a TOPSIS machine tool selection model. In the model, the fuzziness level is presented in terms of the spread (the difference between the lower and upper values) and shift of the mean of the fuzzy numbers. Various cases were developed by increasing the spread and shifting the mean of fuzzy numbers for criteria weights. The benefit, which is defined in terms of the ranking differences, is measured as the statistical significance of the difference between the ranking obtained with the ranking obtained using crisp numbers.

Fig. 2 clearly shows that as the fuzziness level increases, the similarity of the ranking decreases and the benefit of using fuzzy numbers instead of crisp ones increases. However, the calculated Z-values clearly show that the ranking differences are not statistically significant even when the level of fuzziness increased. It can be concluded that when the fuzziness level is low and especially when the mean value of the fuzzy number can be approximately guessed, the benefit of using fuzzy numbers is minimal. Furthermore, if the user is not sure about the mean and spread of the values of a few criteria, he or she can safely guess them as crisp values. On the other hand, in situations in which fuzziness levels are high, although Z-values are still high, the differences in the rankings are high enough to justify the usage of fuzzy numbers instead of the crisp ones.

Appendix A. Fuzzy TOPSIS method

In the fuzzy TOPSIS procedure, the criteria weights ($\tilde{w}_j$; $j=1, 2, \ldots$, number of criteria) and characteristic values of machine tools at criteria (x_{ij}; $i=1, 2, \ldots$, number of machine tools (m), $j=1, 2, \ldots$, number of criteria (n) are inputs and placed in matrix form (Sen and Yang, 1998; Chen, 2000) as shown in Step 1.

Step 1: Inputs are expressed in matrix format as;

$$D = \begin{bmatrix} x_{11} & x_{12} & \cdots & x_{1n} \\ x_{21} & x_{22} & \cdots & x_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ x_{m1} & x_{m2} & \cdots & x_{mn} \end{bmatrix} \quad (1)$$

$$\tilde{W} = [\tilde{w}_1, \tilde{w}_2, \ldots, \tilde{w}_n] \quad (2)$$

$\tilde{w}_j$ are fuzzy numbers and can be described by trapezoidal fuzzy numbers as $\tilde{w}_j = (w_{j1}, w_{j2}, w_{j3}, w_{j4})$ (Chen, 2000).

Step 2: The normalized decision matrix is constructed using Eq. (3) (Sen and Yang, 1998; Byun and Lee, 2004).

$$r_{ij} = \frac{x_{ij}}{\sqrt{\sum_{i=1}^{n} x_{ij}^2}} \quad (3)$$

Step 3: The weighted normalized decision matrix is,

$$\tilde{V} = [\tilde{v}_{ij}]_{m\times n}, \quad i = 1, 2, \ldots, m, \quad j = 1, 2, \ldots, n \quad (4)$$

where,

$$\tilde{v}_{ij} = r_{ij} \otimes \tilde{w}_j \quad (5)$$

Step 4: Each fuzzy number is defuzzified using Eq. (6). For a trapezoidal fuzzy number $\tilde{v}_{ij} = (a_{ij}, b_{ij}, c_{ij}, d_{ij})$ its defuzzification value is defined to be

$$v_{ij} = \frac{a_{ij} + b_{ij} + c_{ij} + d_{ij}}{4} \quad (6)$$

and defuzzified weighted normalized matrix determined as $V = [v_{ij}]_{m\times n}$, $i=1, 2, \ldots, m$, $j=1, 2, \ldots, n$ (Byun and Lee, 2004).

Step 5: The ideal solution, A^* ($A_i^*; i = 1, 2, \ldots, N$), is made of all the best performance scores and the negative-ideal solution, A^- ($A_i^-; i = 1, 2, \ldots, N$), is made of all the worst performance scores at the measures in the defuzzified weighted normalized decision matrix. They are calculated using Eqs. (8) and (10). In these equations, the measures can be divided into two classes: the first is of an input or cost nature, so that smaller performance scores at these measures are preferred; the second is of an output or benefit nature and larger performance scores at these measures are preferred (Sen and Yang, 1998; Byun and Lee, 2004).

$$A^* = (v_1^*, v_2^*, \ldots, v_n^*) \quad (7)$$

$$\overset{*}{v}_j = \left\{ \left(\max_i \ v_{ij} | j \in J \right) \quad i = 1, \ldots m \right\} \quad (8)$$

$$A^- = (v_1^-, v_2^-, \ldots, v_n^-) \quad (9)$$

$$\bar{v}_j = \left\{ (\min_i v_{ij} | j \in J), \quad i = 1, \ldots, m \right\} \quad (10)$$

Step 6: The distance of a alternative i to the ideal solution (d_i^*), and from the negative ideal solution (d_i^-) are calculated using Eqs. (11) and (12) (Sen and Yang, 1998; Byun and Lee, 2004).

$$d_i^* = \sqrt{\sum_{j=1}^{n} (v_{ij} - \overset{*}{v}_j)^2} \quad i = 1, 2, \ldots m; j = 1, 2, \ldots n \quad (11)$$

$$d_i^- = \sqrt{\sum_{j=1}^{n} (v_{ij} - \bar{v}_j)^2} \quad i = 1, 2, \ldots m; j = 1, 2, \ldots m \quad (12)$$

Step 7: The ranking score (C_i^*) is calculated using Eq. (13) (Sen and Yang, 1998; Byun and Lee, 2004).

$$C_i^* = d_i^- / \left(d_i^- + d_i^* \right), \quad i = 1, 2, \ldots m \quad (13)$$

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